

The Distribution of Wealth with Uncertain Income

Ian Irvine and Susheng Wang*

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Abstract

It is well known that an uncertain income stream will induce individuals to save if they are forward looking. In this paper we investigate the consequences of such saving upon the distribution of wealth among households. The inquiry is motivated by the fact that both the dispersion of the U.S. wealth distribution and the variance of transitory income have increased in the recent past. Surprisingly, we find that an increase in earnings uncertainty is more likely to decrease than increase the inequality in wealth holdings.

*The authors are professors at Concordia University, Montreal, Canada, H4X 1T9, and Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, respectively. Financial support from the Social Sciences and Humanities Research Council of Canada and the Research Grant Council of Hong Kong is gratefully acknowledged. The authors benefitted from the suggestions of seminar participants at Concordia University and the University of Colorado at Boulder. They also benefitted from the extensive comments of two referees. Remaining errors are the responsibility of the authors.

1. Introduction

Probabilistic events influence both the accumulation and the distribution of wealth. For example, an unknown date of death, in the absence of full insurance, will influence the rate at which the elderly decumulate their wealth. Or an income stream with a random component will induce prudent individuals to save in a way which differs from when the income stream is known with certainty.

Earnings distributions have become more extended in most economies in the last two decades, and an enormous literature details the many possible reasons for this (e.g. Levy and Murnane, 1992 or Gottschalk and Smeeding, 1997). Among the proposed reasons has been an increase in the dispersion of the random or transitory component of earnings, as documented in Gottschalk and Moffitt (1994). At the same time, the most recent data available for the United States indicate that the distribution of wealth has become significantly more unequal (Wolff, 1998).

The purpose of this paper is to investigate whether there may be a connection between these two observations: how does the variance of the random component of the earnings stream affect the distribution of wealth among households, and how do changes in that variance translate into changes in the dispersion of the stock of wealth? We address this by building a model of utility maximizing agents with different endowments, in an overlapping generations framework, and then perturb the equilibrium distribution through changes in the variance of the earnings streams.

To preview our findings: the effect on the wealth distribution of changes in the variance of income shocks depends upon how individuals plan for the future, and upon the time properties of the process which generates the shocks. In particular, more randomness does not necessarily increase the inequality of the wealth distribution: if individuals plan for income uncertainty over their lifecycle, increased randomness can reduce inequality in the wealth distribution.

The intuition behind this somewhat surprising result has two parts: In lifecycle models individuals reach maximal wealth levels around the age of retirement - at which point earnings uncertainty becomes negligible. The desired asset path of a young person depends, *inter alia*, on a retirement-age goal and on the anticipated earnings shocks *en route*. If earnings uncertainty increases, young workers should save more in the early phase of the lifecycle in order to protect their consumption levels from becoming too prone to shocks - their buffer wealth stocks should be higher at a young age. But such earlier accumulation will decrease the difference between the younger, generally lower-wealth, individuals and the older, generally higher-wealth, individuals.

The reduction in wealth inequality between the young and old is just half of the story, however, because an increase in earnings uncertainty also affects the distribution of wealth within a given age cohort. We show in section 5 that this within-cohort effect of increased uncertainty can compound the between-cohort effect. The reason is that inequality measures are usually calculated with reference to a measure of central tendency. For example, the logarithmic variance, the coefficient of variation, the mean logarithmic deviation (or the Theil-Bernoulli family of which it is a member) all involve the mean or some transformation of the mean. Consequently, the effect of an increase in the randomness of earnings on the wealth distribution within a cohort depends both upon the changed wealth dispersion within the cohort and also upon the change in the mean or its transform. It is quite possible for both the mean (or its transform) and the dispersion to increase and at the same time have the inequality measure decline. This is the outcome we encounter in our simulations.

In addition to investigating wealth accumulation behavior over time, we also examine the age profiles of inequality for earnings, income and consumption. Shorrocks(1975), Greenwood (1987) and Japelli (1995) report, broadly, that wealth inequality declines with cohort age to the point of retirement, but increases thereafter. Deaton and Paxson (1994) report that, for the three economies they studied - the U.K., the U.S. and Taiwan - the age profiles of inequality for consumption, earnings and income have the opposite pattern. The profiles which emerge from our model mirror these findings remarkably well.

Section 2 of the paper contains a brief review of the relevant literature. In section 3 the lifecycle model which we use to analyze the wealth distribution is developed. Section 4 deals with parameterization and measurement issues. The results of the model and the various simulations we perform are presented in section 5. Conclusions are offered in the final section. Since the derivations of the equations of motion are quite long, they are placed in a separate appendix, as are some of the other details.

2. Wealth Inequality

The effect of income uncertainty on savings and accumulation behavior has been the subject of several recent investigations (*e.g.* Browning & Lusardi, 1996, Caballero, 1991, or Carroll, 1992, for example). Most have argued that it exerts a significant influence on the stock of wealth, but there has been little work on how such uncertainty shapes the distribution of wealth. The two leading explanations for the shape of the wealth distribution are the lifecycle model and intergenerational transmissions.

If income is assumed to be known with certainty, Atkinson (1971) argued that, in a pure lifecycle model in which identical households save for a retirement period, the top decile of the wealth distribution could be expected to own between one fifth and one quarter of total wealth. The households in this decile would be those about the retirement age. In contrast, data on wealth holdings indicate that the top decile in most developed economies owns about 60% of total wealth and the present number in the U.S. is about 70%. Second, a very significant degree of dispersion characterizes the distribution of wealth *within* age cohorts. Atkinson infers that inequality in inheritances is important in explaining (in particular the upper tail of) the distribution.

The effect of intergenerational transmissions on the stock of wealth was found to be enormous in the work of Kotlikoff and Summers (1981) and Kotlikoff (1988). Gale and Scholz (1994) subsequently argued, using data on gifts *inter vivos*, that while the effect may not be as great as proposed by Kotlikoff, it is certainly much greater than claimed by, for example, Modigliani (1988). Intergenerational transmissions may be unintended (if aging individuals are prudent, and in the absence of perfect annuity markets); they may be due to altruism or premature death. Altruistic transmissions may also be caused by generational income uncertainty, as developed by Becker and Tomes (1986). In this case, parents care about their offspring, and if the earnings of the latter are uncertain, a wealth stock serves to moderate the consumption flows of those generations in a dynasty which have unusually high or low earnings¹.

The work closest to ours, in terms of methodology and objectives, is that of Vaughan (1988). He develops a continuous-time model in which risk-averse agents derive utility from consumption and the utility of their heirs. The savings function has a stochastic rate of return on a composite asset made up of human and non-human capital. Time of death is also uncertain. Some individuals simply inherit their parents' wealth, others are born without any (primogeniture), and this process generates a Pareto distribution of wealth, where the coefficient in the distribution function depends upon the model's parameters. From our standpoint, the main result is that an increase in the variance of the income stream increases inequality if the degree of risk aversion exceeds unity.

The approach of the present paper is different. We build a multi-period, discrete-time, finite-horizon, overlapping-generations model in order to disentangle the between-cohort and the within-cohort effects of stochastic incomes.

One of the reasons why the role of income uncertainty within the lifecycle on the

¹The influence of different tastes, uncertainty about the time of death, the distribution of income and inheritances, assortive mating, and imperfect capital markets have been explored by Davies (1981, 1982), Blinder (1974, 1976), Flemming (1979) and Wolfson (1979) among others.

wealth distribution has not been explored to a greater extent is that modelling approaches are not very tractable. The recent literature in this area has focussed either upon numerical optimization or upon solving very specific stochastic optimization problems. For example, Hubbard, Skinner and Zeldes (1995), Skinner (1988), Caballero (1991)², and more recently De Nardi (2000), Huggett (1996) and Quadrini and Rios-Rull (1997)

The framework we use has an explicit retirement period, for which agents save during their working life. The time of death is assumed unknown - thus giving rise to bequests, and earnings are stochastic. While individuals are assumed to have the same tastes (utility functions), they are distinguished by different inheritances and different skills, implying that wealth within any age cohort will be unequally distributed. The discrete nature of the model means that wealth inequality can be decomposed into between-cohort differences and within-cohort differences. Correspondingly, a decomposable measure of inequality - the coefficient of variation squared - can be used to analyze the results (Jenkins, 1991, or Davies and Hoy, 1994).

Our generations are linked through unintentional bequests, implying no altruistically motivated bequests (Tachibanaki, 1994). Nonetheless, unintentional bequests can be substantial and unequally distributed, even though utility does not depend upon the income or utility of offspring. Finally, any meaningful framework used to examine the wealth distribution should incorporate a pure retirement phase - one for which accumulation during the working phase takes place. Yet many models in which the income process is stochastic do not incorporate this, because a retirement phase represents a discontinuity in the income process.

3. The Model

3.1. The Setup

Consider an overlapping generations economy in which each agent can live for a maximum of $T + N$ periods. Individuals are distinguished at their economic birth both by ability (and hence initial earnings) and inheritance. They have the same preferences and their income stream is generated by the same stochastic process. These individuals may die at the end of any particular period with probability $1 - p$ from an *accidental death*. Individuals who survive to period $T + N$ die of a *natural death* at the end of

²The models of Skinner (1988) and Caballero (1991) imply that income uncertainty could account for a substantial proportion of private wealth. Guiso, Jappelli and Terlizzese (1992) and Irvine and Wang (1994, 2000) dispute these findings.

that period. The population size is normalized at 1 for each period. Accordingly, the number of individuals dying accidentally in any period is $1 - p$, and the number who have a natural death in any period can be shown to be $\frac{1-p}{1-p^{T+N}}p^{T+N}$, which equals the number who survive to the natural life span $T + N$. The number of births in each period is $\frac{1-p}{1-p^{T+N}}$, which equals the sum of accidental and natural deaths.

The income process for any individual with initial income Y_0 is

$$Y_t = \begin{cases} Y_0 + \xi_t, & t \leq T \\ \xi_t, & t \geq T + 1, \end{cases}$$

where $\{\xi_t\}_{t=0}^{T+N}$ is a random walk:

$$\xi_{t+1} = \xi_t + \varepsilon_{t+1},$$

with $\xi_0 = 0$, and $\{\varepsilon_t\}_{t=1}^{T+N}$ is normally and independently distributed:

$$\begin{aligned} \varepsilon_t &\sim N(0, \sigma_1^2), & \text{for } t \leq T, \\ \varepsilon_t &\sim N(0, \sigma_2^2), & \text{for } t > T \end{aligned}$$

Thus, at time $t = 0$

$$E(Y_t) = Y_0 \quad \text{for } t \leq T; \quad E(Y_t) = 0 \quad \text{for } t \geq T + 1.$$

Tastes are defined by the exponential utility function which has constant absolute risk aversion (CARA). There are two stages in life: work and retirement. They are distinguished by a decline in expected income of Y_0 at the point of retirement and also a reduction in the income variance. An individual faces the following maximization problem:

$$\left\{ \begin{array}{l} V(A_0) = \max E_0 \sum_{t=1}^{T+N} -\frac{1}{\theta} e^{-\theta c_t} \left(\frac{p}{1+\delta} \right)^t \\ \text{s.t. } A_t = (1+r)A_{t-1} + Y_t - C_t, \\ A_{T+N} \geq 0, \text{ given } A_0, \end{array} \right. \quad (3.1)$$

where E_t is the expectations operator conditional on information available at time t , θ is both the coefficient of absolute risk aversion and the measure of prudence³, C_t

³Kimball (1990) develops the concept of prudence, which is really an application of the Arrow-Pratt measures of risk aversion to dynamic problems. With the exponential utility function, the prudence measure $(-u'''/u'')$ is the same as the Arrow-Pratt measure $(-u''/u')$. They are each θ in our specification.

is consumption, A_t is nonhuman wealth, r is the interest rate, and δ is the rate of time preference.

The initial wealth A_0 is determined by the intergenerational equilibrium condition that the wealth stock of those dying in any period is passed on to those who are born in that period⁴. The form of the distribution function for A_0 is defined in the following section.

To facilitate the development of the results we use the following notation.

$$\alpha \equiv \frac{1}{1+r}, \quad \beta \equiv p \frac{1+r}{1+\delta}, \quad \Gamma_1^* \equiv \frac{1}{2} \theta \sigma_2^2 + \frac{1}{\theta} \ln \beta, \quad \Gamma_2^* \equiv \frac{1}{2} \theta \sigma_2^2 + \frac{1}{\theta} \ln \beta.$$

Denote

$$\begin{aligned} \bar{t}_p &\equiv \frac{1-p}{1-p^{T+N}} \sum_{t=1}^{T+N} t p^{t-1} = \frac{1}{1-p} - \frac{(T+N)p^{T+N}}{1-p^{T+N}}, & \bar{t}_\alpha &\equiv \frac{1}{1-\alpha} - \frac{(T+N)\alpha^{T+N}}{1-\alpha^{T+N}}, \\ \text{pop}_p &\equiv \frac{1-p}{1-p^{T+N}} \sum_{t=1}^T p^{t-1} = \frac{1-p^T}{1-p^{T+N}}, & \text{pop}_\alpha &\equiv \frac{1-\alpha^T}{1-\alpha^{T+N}}. \end{aligned}$$

\bar{t}_p is the average age of the population and pop_p is the size of the working population.

3.2. The Optimal Solution

As illustrated in the appendix, solving (3.1) gives the individual wealth profile:

$$\begin{aligned} A_t &= \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} A_0 + \frac{(1-\alpha^N)(\alpha^{T-t}-\alpha^T)}{1-\alpha^{T+N}} \frac{Y_0}{r} + \frac{\alpha^{T-t}-\alpha^T}{1-\alpha^{T+N}} \left(\frac{1-\alpha^N}{1-\alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{r} \\ &\quad + \left[t - \frac{\alpha^{T+N-t}-\alpha^{T+N}}{1-\alpha^{T+N}} (T+N) \right] \frac{\Gamma_1^*}{r}, \quad \text{for } t \leq T; \end{aligned} \quad (3.2a)$$

$$\begin{aligned} A_t &= \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} A_0 + \frac{(1-\alpha^T)(1-\alpha^{T+N-t})}{1-\alpha^{T+N}} \frac{Y_0}{r} + \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} \left(\frac{1-\alpha^T}{1-\alpha} - T \right) \frac{\Gamma_2^* - \Gamma_1^*}{r} \\ &\quad + \left[t - \frac{\alpha^{T+N-t}-\alpha^{T+N}}{1-\alpha^{T+N}} (T+N) \right] \frac{\Gamma_2^*}{r}, \quad \text{for } t \geq T, \end{aligned} \quad (3.2b)$$

and the maximum expected utility:

$$V(A_0) = -\frac{1-\alpha^{T+N}}{\theta r} e^{-\theta \left[\frac{r}{1-\alpha^{T+N}} A_0 + \frac{1-\alpha^T}{1-\alpha^{T+N}} Y_0 + \left(T\alpha^{T+N} - \frac{1-\alpha^T}{1-\alpha} \right) \frac{\Gamma_1^* - \Gamma_2^*}{1-\alpha^{T+N}} - \bar{t}_\alpha \Gamma_2^* \right]}. \quad (3.3)$$

⁴The assumption that the inheritance is received at the beginning of the economic life can be relaxed without affecting the closed form nature of the solutions.

The budget constraint then gives the consumption:

$$\begin{aligned}
C_t &= Y_t - Y_0 + \frac{r}{1 - \alpha^{T+N}} A_0 + \frac{1 - \alpha^T}{1 - \alpha^{T+N}} Y_0 + \left(T - \frac{1 - \alpha^T}{1 - \alpha} \right) \frac{\Gamma_1^* - \Gamma_2^*}{1 - \alpha^{T+N}} + (T - \bar{t}_\alpha) \Gamma_2^* + (t - T) \Gamma_1^*, \quad t \leq T, \\
C_t &= Y_t + \frac{r}{1 - \alpha^{T+N}} A_0 + \frac{1 - \alpha^T}{1 - \alpha^{T+N}} Y_0 + \left(T - \frac{1 - \alpha^T}{1 - \alpha} \right) \frac{\Gamma_1^* - \Gamma_2^*}{1 - \alpha^{T+N}} + (T - \bar{t}_\alpha) \Gamma_2^* + (t - T) \Gamma_2^*, \quad t > T.
\end{aligned} \tag{3.4}$$

The savings function in each period is defined by $S_t^* = Y_t - C_t^*$ and can be obtained immediately from the consumption function.

3.3. Equilibrium

Suppose the population is randomly distributed over the space for (A_0, Y_0) and ε_t is uncorrelated with A_0 nor Y_0 for any t . Let f be the joint density function of the random variables (A_0, Y_0) for the newborn, with $\int f(a, y) da dy = 1$. Denote

$$\sigma_{A_0}^2 \equiv \text{var}(A_0), \quad \sigma_{Y_0}^2 \equiv \text{var}(Y_0), \quad \mu_{A_0} \equiv E(A_0), \quad \mu_{Y_0} \equiv E(Y_0).$$

Since the size of the newborn cohort is $\frac{1-p}{1-p^{T+N}}$, the number of newborn with endowment near $A_0 = a$ and $Y_0 = y$ is $\frac{1-p}{1-p^{T+N}} f(a, y) \Delta a \Delta y$, it follows that

$$\text{total wealth endowment} = \frac{1-p}{1-p^{T+N}} \int a f(a, y) da dy = \frac{1-p}{1-p^{T+N}} E(A_0),$$

where the expectation is based on the population density function f .

The first intergenerational equilibrium condition is that total bequests equal total inheritances:

$$(1-p)W^* = \frac{1-p}{1-p^{T+N}} E(A_0), \quad \text{or} \quad \mu_{A_0} = (1-p^{T+N})W^*. \tag{3.5}$$

$$\text{with} \quad W^* = \frac{1-p}{1-p^{T+N}} \sum_{t=1}^{T+N} p^{t-1} E(A_t^*), \tag{3.6}$$

where the expectation is over Y_0 and A_0 jointly.

The second intergenerational equilibrium condition is that the dispersion of the inheritances distribution be the same as that of the bequests distribution. Furthermore, since we use the coefficient of variation as the measure of dispersion, and since this

is population invariant, it follows that these distributions have the same coefficient of variation as the wealth distribution of the whole population⁵.

It will prove convenient to rewrite equation (3.2) in the form

$$A_t^* = a_t A_0 + b_t Y_0 + c_t. \quad (3.7)$$

Substituting this into (3.6), and using (3.5) we can rewrite the equilibrium W^* as

$$\begin{aligned} W^* = & \frac{1 - \alpha^{T+N}}{\alpha^{T+N} - p^{T+N}} \left[(\text{pop}_p - \text{pop}_\alpha) \frac{\mu_{Y_0}}{r} + (\bar{t}_\alpha - \bar{t}_p) \frac{\Gamma_1^*}{r} \right] - N \frac{\Gamma_2^* - \Gamma_1^*}{(1 - p^{T+N}) r} \\ & + \left[\frac{\alpha^T - \alpha^{T+N}}{1 - \alpha} - \frac{(1 - \alpha^{T+N})(p^T - p^{T+N})}{(1 - p)(1 - p^{T+N})} \right] \frac{\Gamma_2^* - \Gamma_1^*}{(\alpha^{T+N} - p^{T+N}) r}. \end{aligned} \quad (3.8)$$

Before examining the wealth distribution which emerges from this framework, several points should be noted. First, the permanent nature of the income shocks implies that consumption adjusts fully in each period, rather than with a lag⁶. The corollary of this is that while consumption is stochastic, saving is not, implying that wealth W^* in (3.8) is non-stochastic, in the sense of being independent of the actual realizations of Y_t . Second, even though there is no explicit bequest motive, wealth is passed between generations. Consequently, while inheritances are received because of an uncertain date of death, the *amount* of such inheritances will depend upon the degree of risk aversion, uncertainty about death and income, etc. Third, insurance is available neither for income nor lifespan uncertainty. This absence may be attributable to moral hazard, and is standard in models of income uncertainty. Fourth, as is known for models based upon the CARA utility function, it is possible for negative consumption to materialize for individuals who experience a series of negative income shocks. Technically this is because the marginal utility of consumption is not infinite at a zero level of consumption. The imposition of a non-negativity constraint changes the solution for the equations of motion in such a way that the model can no longer be solved analytically. However, in this particular form of the model - where individuals start their economic life with an ‘inheritance’, our simulations indicate that the incidence of such negative consumption is exceedingly low: the inheritance provides a buffer wealth stock which insulates them. Accordingly, the required series of negative shocks for consumption to be negative at some point in the lifecycle has an extremely low probability. Fifth, the

⁵This follows from the fact that a proportion $(1 - p)$ of each cohort dies every period, and therefore $(1 - p)$ of every cohort’s wealth goes to bequests/inheritances. Accordingly the distribution of bequests/inheritances is a scalar multiple of the whole economy’s wealth distribution.

⁶Note that while the shock in our model is permanent in the level of income, this specification is consistent with Gottschalk and Moffitt (1994) whose shock is transitory in the growth rate of income.

model permits a change the variance of earnings/income at the point of retirement, including the option of setting it equal to zero. Last, some individual wealth profiles are illustrated in the final section of each panel in figure 1. For most parameter value sets, wealth reaches a peak at the point of retirement.

4. Measurement and Parameterization

4.1. Inequality Measurement

Observed wealth inequality within cohorts is always high and varies with the age of the cohort. This suggests that the coefficient of variation CV_T , defined by

$$CV_T \equiv \frac{1}{W^*} \sqrt{\sum_{t=1}^{T+N} \frac{1-p}{1-p^{T+N}} p^{t-1} E[(A_t^* - W^*)^2]}, \quad (4.1)$$

would be a productive measure of inequality because its square is decomposable into between-cohort CV_B and within-cohort CV_W components. Moreover, its square is a member of the generalized entropy family of inequality measures (Jenkins, 1991), it is additively decomposable, mean invariant, satisfies the principle of transfers and is homogeneous of degree zero in population size. The decomposition for our model is developed in Appendix B and results in

$$CV_B \equiv \frac{1}{W^*} \sqrt{\sum_{t=1}^{T+N} \frac{1-p}{1-p^{T+N}} p^{t-1} [E(A_t^*) - W^*]^2}, \quad (4.2)$$

$$CV_W \equiv \frac{1}{W^*} \sqrt{\sum_{t=1}^{T+N} \frac{1-p}{1-p^{T+N}} p^{t-1} \text{var}(A_t^*)}. \quad (4.3)$$

The square of the CV_T satisfies decomposability, because $(CV_T)^2 = (CV_B)^2 + (CV_W)^2$.

4.2. Parameterization

The basic set of parameter values is governed by available empirical evidence. Browning and Lusardi (1996) show that the bulk of saving in most developed economies takes place late in the working phase of the lifecycle. Accordingly, and consistent with Carroll's view (1992) that individuals tend to be impatient, we set the rate of time preference above the interest rate ($r = 2\%$ and $\delta = 4\%$), so that most saving takes

place at the appropriate time. The difference between r and δ determines the growth in the consumption stream. From (3.4) it is straightforward to show that for $t < T$

$$\Delta C_t^* = \varepsilon_t + \Gamma_1^* \simeq \varepsilon_t + \frac{1}{2}\theta\sigma_1^2 + \frac{r - \delta}{\theta}$$

Impatience is captured in the final term, and risk and the degree of aversion to risk in the middle term⁷. We normalize the mean of the earnings distribution $E(Y_0) = 100$ and set the prudence coefficient θ at 2%.

Our primary objective is to examine how the wealth distribution responds to variations in the stochastic element in earnings. MaCurdy (1982) suggests a value for $\sigma/Y = 0.10$, although Guiso *et al* (1991) suggest a value as low as 0.02. Our base value is set at 0.05 for the working period and is reduced to zero for the retirement period⁸. The expected length of the economic life is set equal to 50 years, with a working life of 40. When the lifetime is uncertain there is an infinite number of combinations of p and $T + N$ which will give an expected lifetime of 50 years; $T + N = 57$ and $p = 0.99523$ is one such combination (suggested by Caballero).

This set of values generates a wealth to *GDP* ratio between 4 and 5, which is a typical value for a developed economy.

Finally we require some assumptions on the dispersion of inheritances A_0 and human capital Y_0 , and the correlation between them. We impose a coefficient of variation on Y_0 of unity. Since the *CV* of bequests equals the *CV* of the wealth distribution, as described above, the value of $CV(A_0)$ which is consistent with a steady state is obtained by numerically iterating for the equilibrium solution. Lastly, we specify a positive value for the correlation coefficient linking the distributions of wealth and human capital: individuals dying with more wealth are assumed to have descendants with relatively more human capital (Gale and Scholz, 1994).

⁷Since we cannot incorporate a trend in Y_t if we are to obtain explicit equations of motion, the parameterizations could be considered to be net of any actual trend observed in real processes. Thus, while savings are generated late in the working life by having a declining consumption stream, this consumption stream would not necessarily decline relative to an upwardly trending income process.

⁸In an earlier version of this paper we considered the possibility of two types of income uncertainty: uncertainty in working-life earnings, and also in the demands which may be placed upon their an individual's resources in retirement - for example unpredictable health conditions, better or poorer than the norm. We therefore permitted the variance of the 'earnings' stream to be non zero post-retirement. The results we obtained from this exercise were very similar to what is presented in this version of the paper, table 1. Regardless of how one might view the random component in the retirement period, our simulations focus upon varying σ_1 only.

5. Results and Simulations

The results are presented in figure 1 and table 1. Each panel contains a description of within-cohort variation over time $CV_W(t)$ - as defined by appendix equation (B.3); between-group variation over time $CV_B(t)$ as defined by equation (B.4), and the asset path A_t for a household with average human capital $Y_0 = 100$, as defined by equation (3.2). Summary numerical values are given in the corresponding rows of table 1. Panel A defines the solution for the base-case parameterization defined above.

The asset paths A_t are particularly important in interpreting the results: significant changes in inequality can arise as a result of changes in a desired asset path - which appears in the denominator of the inequality measure, even if accompanied by small changes in the dispersion measure - the standard deviation in the numerator. The base-case equilibrium solution to the model yields a coefficient of variation CV_{A_0} , obtained from eq. (4.1), of 2.34. The relative importance of CV_B and CV_W is given in table 1. In all simulations the within-group components are larger than the between-group, reflecting actual patterns (Greenwood, 1987).

The summary statistic for the overall degree of inequality in the economy in table 1 and at the head of each panel is obtained from the relation $(CV_T)^2 = (CV_B)^2 + (CV_W)^2$, where the components are defined by equations (4.1) - (4.3). We note also that $CV_{A_0} = CV_T$.

5.1. Within-Group Variation

Consider initially panel A. The value of $CV_{W,t}$ in the early years of any cohort's life is determined primarily by the distribution of A_0 . This is because the amount of wealth saved, and consequently the contribution of this to the dispersion in wealth, is yet small. As a cohort moves through time this initial value, $CV(A_0)$, becomes less important relative to the variation in human capital and the wealth accumulation pattern of individuals within that cohort. In all cases the variance (and standard deviation) of the wealth holdings of a cohort increases over the working life, as does the average asset holding - as shown in the final part of each panel. It is their relative rates of increase which determine the path of $CV_{W,t}$. The pattern of $CV_{W,t}$ in panel A indicates that the standard deviation initially rises more steeply than average asset holdings within a cohort, but then the rate of accumulation surpasses the rate of growth in the standard deviation until the age of retirement - giving rise to the decline in the $CV_{W,t}$ to that point.

Correspondingly, the standard deviation of a cohort's wealth falls throughout retirement, though at a slightly slower rate than average assets, giving rise to a mild increase in the value of $CV_{W,t}$ during that period.

Panels *B* through *E* portray the results for each simulation. Our primary interest is in the effects of a change in the variance of the income process. Panel *B* indicates that a reduction in this value during the working life from 5% to 3% increases the degree of inequality in the wealth distribution, and the effect works in the same direction both between cohorts and within cohorts. The within-cohort effect of such a decrease is twofold. It reduces the variance of wealth holdings for a cohort - as expected, but also reduces the mean asset holdings by a greater degree, with the net effect that the within-cohort inequality increases. The key to this result is simple: inequality measures generally reflect dispersion relative to the mean or a transformation of the mean. If the dispersion is a function of the second moment (as is the standard deviation), but the inequality measure is a function of this relative to the mean, then decreases in dispersion are consistent with increases in inequality. As indicated in row 1 of the table, the CV_W increases from 2.21 to 2.54.

Parenthetically, we note that, as indicated by eq (B.1), the reduction in $Var(A_t)$ comes in part through a reduction in the variance of inheritances, which in turn is due to the lower overall wealth levels in the economy.

5.2. Between-Group Variation

The measure $CV_{B,t}$, given in the middle frame of each panel is a measure of the deviation of a cohort's wealth from the overall wealth of the economy as that cohort moves through time (B.4). At the extremes of the lifecycle - when it has negligible wealth - the cohort extends the wealth distribution in the economy, whereas when it has wealth similar to the economy's average, it contributes little to the overall dispersion between groups. This is mirrored by the high values at the extremes of the lifecycle and very low values in the middle years. The slight 'W' shape in the middle is explained by the behavior around the peak in the asset path: approaching retirement, average asset holdings of a cohort are deviating from the mean for the economy (hence the upward sloping segment in the middle of the between-group function), before returning towards the mean immediately post-retirement (hence the downward sloping segment), and finally deviating significantly from the economy's mean until the time of death.

The simulation involving a reduction in the earnings variance, as indicated in row 2 of the table, increases the between-group variation by inducing younger cohorts to

accumulate less vigorously: they now have less of a need to insulate themselves against bad earnings shocks during the lifecycle, and consequently they build up a smaller buffer stock. This is due to the forward looking behavior of individuals and the permanence of the income shocks. Rather than reacting to unanticipated income shocks, individuals are prudent. As a consequence, relative to the middle aged cohorts, the young now have less wealth. The value of CV_B increases from 0.78 to 0.87.

5.3. Further Simulations

The remaining three panels explore the effect of different parameterizations on the basic set of results.

- Panel *C* indicates that an increase in the risk aversion parameter from $\theta = 2\%$ to $\theta = 3\%$ reduces wealth inequality considerably. Essentially this is the same qualitative result as in panel B - individuals in the younger cohort become more prudent by saving more early in their life.
- Panel *D* explores the effect of changing the relationship between the interest rate and the rate of time preference. Here the rates take on equal values. The result is that a higher interest rate induces a greater accumulation of assets. This is illustrated in the asset path. The numerical outcome of the experiment indicates that this increase in the denominator of the CV overpowers any effects it may have in the numerator.
- Panel *E* indicates that the assumed correlation between inheritances and human capital is relatively unimportant – CV_T declines from 2.34 in the base case to 2.25 when the correlation is reduced from 0.5 to 0.1. This is consistent with Davies (1982) who points out that the effect of the correlation should depend upon the magnitude of inheritances.

5.4. Consumption, Earnings and Income by Cohort

As a means of validating the model which has generated our results, its predictions can be compared with the findings of Deaton and Paxson, who show that within-cohort age inequality increases for consumption, earnings and income for the U.S., the U.K. and Taiwan. Since earnings follow a random walk in our model, it follows immediately that the dispersion of earnings within a cohort will increase over time. As for consumption within a cohort, equation (3.4) suggests that it too should have an increasing degree of inequality since it depends linearly upon income. Intuition would

also suggest that income within a cohort should exhibit increasing dispersion to the degree that earnings are a major component of income.

Using the decomposition of the CV in appendix B, that is (B.3), the age profiles of inequality for earnings, consumption and income are given in the three panels of figure 2. Like Deaton and Paxson these profiles are all increasing to the point of retirement.

6. Conclusion

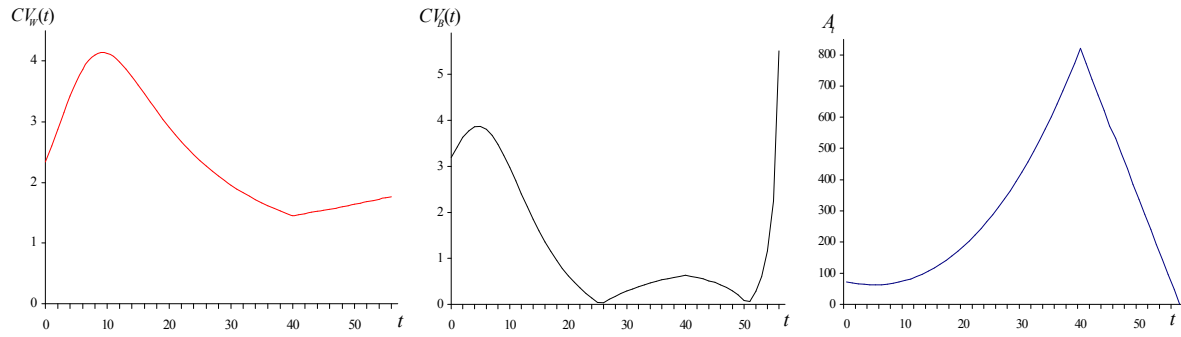
Our primary motivation for examining this question was the observation that both the variation in the transitory component of individual earnings and the dispersion in the wealth distribution have increased in several economies in the last two decades. In the context of a framework where individuals are forward-looking and prudent, our finding is that the influence of income uncertainty on the wealth distribution is not in the direction where intuition would first lead us: there is a good theoretical reason to suppose that the between-cohort effect will be in the opposite direction to the movement in the variance - less (more) uncertainty reduces (increases) the incentive for young people to save and their wealth stock may fall further below (approach more closely) the wealth of those nearer to retirement. Furthermore there is no *a priori* reason as to why the within-cohort effect should move in one direction rather than the other: the outcome will depend upon how the particular specification affects the variation in wealth relative to the desired asset path for a given cohort.

The model is sparse when compared with numerical models where exact solutions to the equations of motion are not required⁹. Yet it replicates many of the stylized facts on income, consumption and wealth: saving is concentrated in the late working life, the variance of the distribution of income and consumption within a cohort increase with time up to the point of retirement, and bequests are unintentional. In addition to the numerical results, the framework provides insights into the complex structure that links a stochastic income flow to the distribution of the stock of wealth.

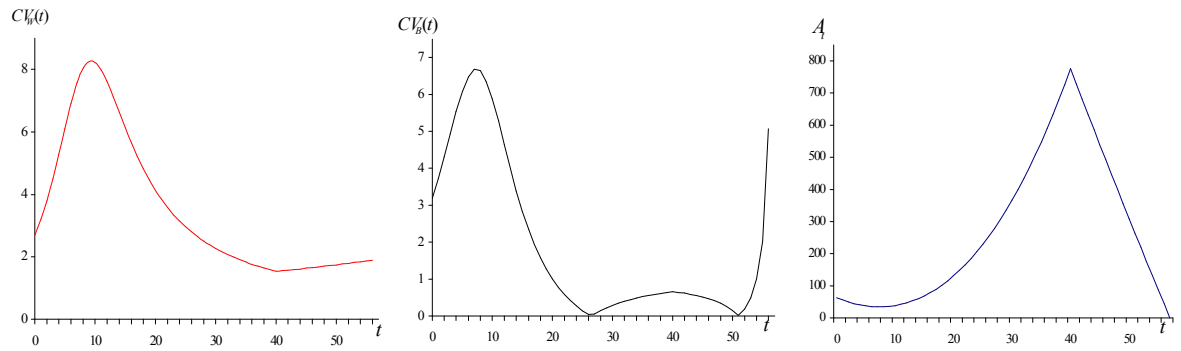
⁹For example, Hubbard, Skinner and Zeldes (1995) are able to generate a significant density of zero-wealth holdings at retirement, by modelling the behavior of high and low ability individuals who are motivated in their wealth accumulation by the presence of social security support in retirement, which is subject to an asset-based means test.

Figure 1

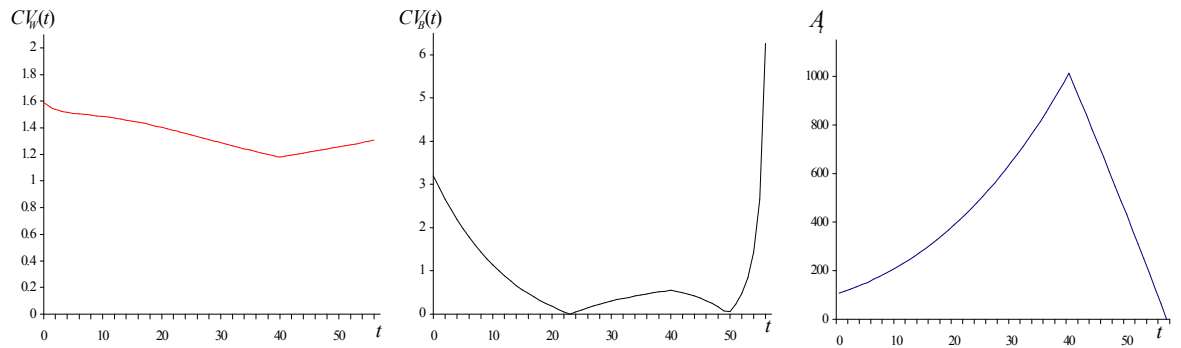
Panel A: $r = 2\%$, $\delta = 4\%$, $\sigma_1 = 5$, $\sigma_2 = 3$, $\theta = 2\%$, $\rho = 0.5$, $CV_{Y_0} = 1$, $CV_{A_0} = 2.31$



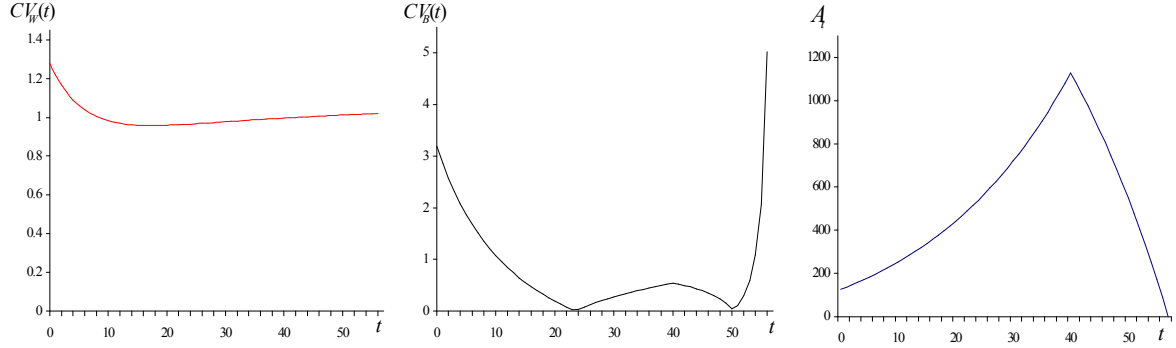
Panel B: $r = 2\%$, $\delta = 4\%$, $\sigma_1 = 3$, $\sigma_2 = 3$, $\theta = 2\%$, $\rho = 0.5$, $CV_{Y_0} = 1$, $CV_{A_0} = 2.64$



Panel C: $r = 2\%$, $\delta = 4\%$, $\sigma_1 = 5$, $\sigma_2 = 3$, $\theta = 3\%$, $\rho = 0.5$, $CV_{Y_0} = 1$, $CV_{A_0} = 1.56$



Panel D: $r = 4\%$, $\delta = 4\%$, $\sigma_1 = 5$, $\sigma_2 = 3$, $\theta = 2\%$, $\rho = 0.5$, $CV_{Y_0} = 1$, $CV_{A_0} = 1.27$



Panel E: $r = 2\%$, $\delta = 4\%$, $\sigma_1 = 5$, $\sigma_2 = 3$, $\theta = 2\%$, $\rho = 0.1$, $CV_{Y_0} = 1$, $CV_{A_0} = 2.22$

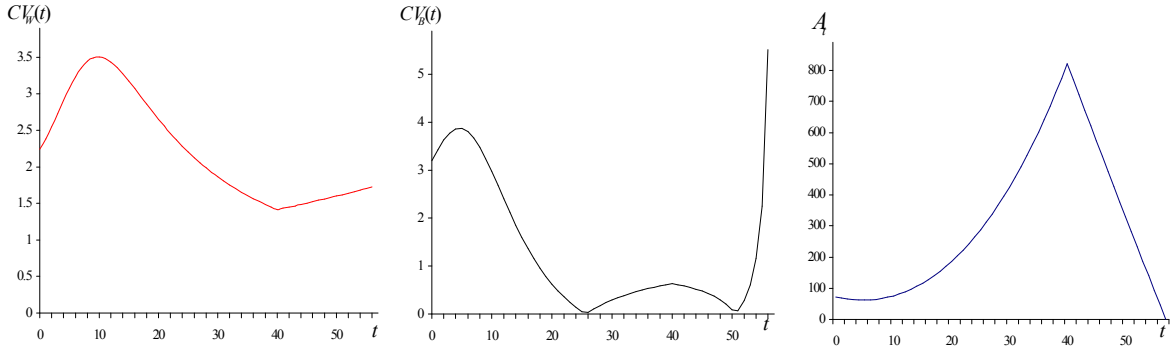
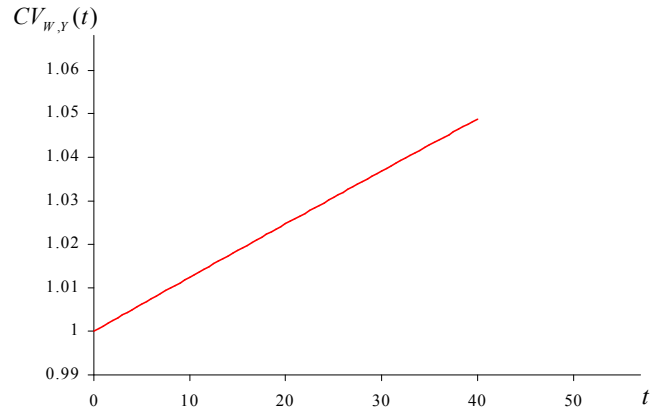


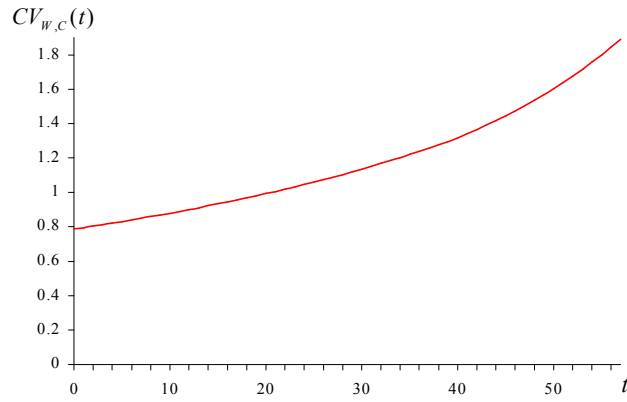
Table 1. Wealth Distribution

Case	Parameterization								Result				
	CV_{A_0}	CV_{Y_0}	σ_1/Y_0	σ_2/Y_0	r	δ	θ	ρ	CV_W	CV_B	CV_T	A_0	W
A	2.31	1	5%	3%	2%	4%	2%	0.5	2.17	0.77	2.31	73.2	306.8
B	2.64	1	3%	3%	2%	4%	2%	0.5	2.49	0.86	2.64	63.8	267.4
C	1.56	1	5%	3%	2%	4%	3%	0.5	1.44	0.60	1.56	110.6	463.5
D	1.27	1	5%	3%	4%	4%	2%	0.5	1.13	0.57	1.27	126.4	529.9
E	2.22	1	5%	3%	2%	4%	2%	0.1	2.08	0.77	2.22	73.2	306.8

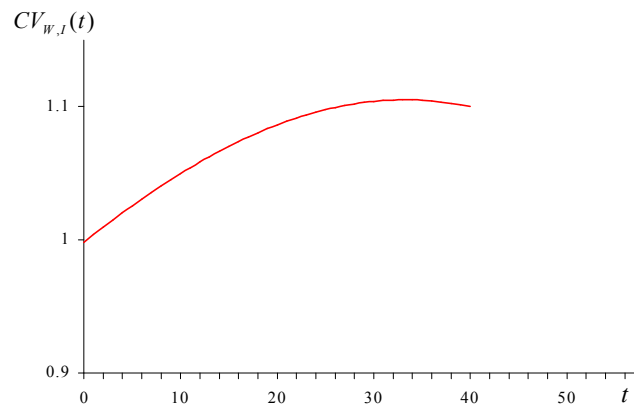
Figure 2.



Within-Group CV for Earnings¹⁰



Within-Group CV for Consumption



Within-Group CV for Income

¹⁰The CV for earnings is truncated at the point of retirement since the expected value of earnings in the denominator is zero after that point. Likewise, the CV for income increases very rapidly since the expected value of the earnings component becomes zero, leaving interest as the only source of income.

Appendix A

A.1. Euler Equation

We now solve problem (2.1) for the Euler equation using backward induction. We will solve it for a general atemporal utility function $u(c)$.

Problem in the retirement stage:

Given the initial wealth A_T , the consumer problem in the retirement stage is

$$\left\{ \begin{array}{l} V_T(A_T) \equiv \max E_T \sum_{t=T+1}^{T+N} -\frac{1}{\theta} e^{-\theta c_t} \left(\frac{p}{1+\delta} \right)^t \\ \text{s.t. } A_t = RA_{t-1} + Y_t - C_t \\ A_{T+N} \geq 0, \text{ given } A_T, \end{array} \right.$$

where $R \equiv 1 + r$. It is a standard recursive problem, and the Euler equation is well known as

$$u'(C_t^*) = R\rho E_{t-1} [u'(C_{t+1}^*)], \quad t \geq T, \quad (\text{A.1})$$

where $\rho \equiv \frac{p}{1+\delta}$. We will now continue backward induction from $t = T$.

Problem at period T :

$$\left\{ \begin{array}{l} V_{T-1}(A_{T-1}) \equiv \max_{c_T} \rho^T u(C_T) + E_{T-1} V_T(A_T) \\ \text{s.t. } A_T = RA_{T-1} + Y_T - C_T, \\ \text{given } A_{T-1}. \end{array} \right.$$

It can be reduced to

$$V_{T-1}(A_{T-1}) \equiv \max_{C_T} \rho^T u(C_T) + E_{T-1} V_T(RA_{T-1} + Y_T - C_T),$$

which gives the first-order condition:

$$\rho^T u'(C_T^*) = E_{T-1} V_T'(A_T^*).$$

The envelope theorem also implies

$$V'_{T-1}(A_{T-1}^*) = R E_{T-1} V'_T(A_T^*).$$

Problem at period $T - 1$:

$$\left\{ \begin{array}{l} V_{T-2}(A_{T-2}) \equiv \max_{C_{T-1}} \rho^{T-1} u(C_{T-1}) + E_{T-2} V_{T-1}(A_{T-1}) \\ \text{s.t. } A_{T-1} = R A_{T-2} + Y_{T-1} - C_{T-1}, \\ \text{given } A_{T-2}. \end{array} \right.$$

It can be reduced to

$$V_{T-2}(A_{T-2}) \equiv \max_{C_{T-1}} \rho^{T-1} u(C_{T-1}) + E_{T-2} V_{T-1}(R A_{T-2} + Y_{T-1} - C_{T-1}),$$

which gives the first-order condition:

$$\rho^{T-1} u'(C_{T-1}^*) = E_{T-2} V'_{T-1}(A_{T-1}^*).$$

The envelope theorem also implies

$$V'_{T-2}(A_{T-2}^*) = R E_{T-2} V'_{T-1}(A_{T-1}^*).$$

The solution:

We can now see a clear pattern. We will generally have

$$\rho^t u'(C_t^*) = E_{t-1} V'_t(A_t^*), \quad (\text{A.2})$$

and

$$V'_{t-1}(A_{t-1}^*) = R E_{t-1} [V'_t(A_t^*)], \quad (\text{A.3})$$

for $t = 1, \dots, T$. (A.2) and (A.3) imply $R \rho^t u'(C_t^*) = V'_{t-1}(A_{t-1}^*)$. Using (A.2) again, we have

$$\rho^{t-1} u'(C_{t-1}^*) = E_{t-2} V'_{t-1}(A_{t-1}^*) = E_{t-2} [R \rho^t u'(C_t^*)].$$

Thus,

$$u'(C_t^*) = R \rho E_{t-1} [u'(C_{t+1}^*)], \quad t \leq T - 1.$$

Combining with (A.1), the Euler equation is thus

$$u'(C_t^*) = R \rho E_{t-1} [u'(C_{t+1}^*)], \quad \text{for all } t. \quad (\text{A.4})$$

A.2. The Optimal Solution — Section 3.2

The Difference Equation for Individual Wealth:

For our utility function $u(c) = -\frac{1}{\theta}e^{-\theta c}$, the Euler equation (A.4) becomes

$$e^{-\theta C_t} = \beta E_{t-1} (e^{-\theta C_{t+1}}), \quad 1 \leq t \leq T + N. \quad (\text{A.5})$$

One can easily verify that the following is a solution for (A.5):

$$\begin{aligned} C_{t+1} - C_t &= \Gamma_1^* + \varepsilon_{t+1}, & \text{for } t < T, \\ C_{t+1} - C_t &= \Gamma_2^* + \varepsilon_{t+1}, & \text{for } t \geq T. \end{aligned} \quad (\text{A.6})$$

By (A.6),

$$\begin{aligned} C_{t+1} - C_t &= \Gamma_1^* + Y_{t+1} - Y_t, & \text{for } t < T, \\ C_{T+1} - C_T &= \Gamma_2^* + Y_0 + Y_{T+1} - Y_T, & \text{for } t = T, \\ C_{t+1} - C_t &= \Gamma_2^* + Y_{t+1} - Y_t & \text{for } t > T, \end{aligned}$$

Then, by the budget constraint, for $t < T$, we have

$$\Gamma_1^* + Y_{t+1} - Y_t = C_{t+1} - C_t = R(A_t - A_{t-1}) + Y_{t+1} - Y_t - (A_{t+1} - A_t),$$

implying

$$A_t - A_{t-1} = \alpha(A_{t+1} - A_t) + \alpha\Gamma_1^*, \quad t < T. \quad (\text{A.7a})$$

Similarly, for $t > T$,

$$A_t - A_{t-1} = \alpha(A_{t+1} - A_t) + \alpha\Gamma_2^*, \quad t > T. \quad (\text{A.7b})$$

For $t = T$, we have

$$\Gamma_2^* + Y_{T+1} - Y_T = C_{T+1} - C_T - Y_0 = R(A_T - A_{T-1}) + Y_{T+1} - Y_T - Y_0 - (A_{T+1} - A_T).$$

Then,

$$A_T - A_{T-1} = \alpha(A_{T+1} - A_T) + \alpha\Gamma_2^* + \alpha Y_0. \quad (\text{A.7c})$$

Individual Wealth:

Then, for $t > T$,

$$A_t - A_{t-1} = \alpha^{T+N-t} (A_{T+N} - A_{T+N-1}) + \Gamma_2^* \sum_{i=1}^{T+N-t} \alpha^i,$$

implying

$$\begin{aligned}
A_{t-1} &= A_t - \frac{\alpha - \alpha^{T+N+1-t}}{1-\alpha} \Gamma_2^* - \alpha^{T+N-t} (A_{T+N} - A_{T+N-1}) \\
&= A_{t+1} - \frac{\alpha - \alpha^{T+N-t}}{1-\alpha} \Gamma_2^* - \alpha^{T+N-t-1} (A_{T+N} - A_{T+N-1}) - \frac{\alpha - \alpha^{T+N+1-t}}{1-\alpha} \Gamma_2^* \\
&\quad - \alpha^{T+N-t} (A_{T+N} - A_{T+N-1}) \\
&= \dots \\
&= A_{T+N} - \Gamma_2^* \sum_{i=t}^{T+N} \frac{\alpha - \alpha^{T+N+1-i}}{1-\alpha} - (A_{T+N} - A_{T+N-1}) \sum_{i=t}^{T+N} \alpha^{T+N-i} \\
&= A_{T+N} - \frac{\alpha \Gamma_2^* (1-\alpha)(T+N) - \alpha - (1-\alpha)t + \alpha^{T+N+1-t}}{1-\alpha} - (A_{T+N} - A_{T+N-1}) \frac{1 - \alpha^{T+N+1-t}}{1-\alpha},
\end{aligned}$$

implying

$$A_t = A_{T+N} - (A_{T+N} - A_{T+N-1}) \frac{1 - \alpha^{T+N-t}}{1-\alpha} - \frac{(1-\alpha)(T+N-t-1) - \alpha + \alpha^{T+N-t}}{1-\alpha} \frac{\alpha \Gamma_2^*}{1-\alpha}.$$

Since $A_{T+N}^* = 0$, we have

$$A_t = \frac{1 - \alpha^{T+N-t}}{1-\alpha} A_{T+N-1} - \frac{(1-\alpha)(T+N-t-1) - \alpha + \alpha^{T+N-t}}{1-\alpha} \frac{\alpha \Gamma_2^*}{1-\alpha}, \quad t \geq T.$$

In particular, for $t = T$,

$$A_T = \frac{1 - \alpha^N}{1-\alpha} A_{T+N-1} - \frac{(1-\alpha)(N-1) - \alpha + \alpha^N}{1-\alpha} \frac{\alpha \Gamma_2^*}{1-\alpha}.$$

The above two imply

$$\begin{aligned}
(1 - \alpha^N) A_t - (1 - \alpha^{T+N-t}) A_T &= \frac{\alpha \Gamma_2^*}{(1-\alpha)^2} \left\{ (1 - \alpha^{T+N-t}) [(1-\alpha)(N-1) - \alpha + \alpha^N] \right. \\
&\quad \left. - (1 - \alpha^N) [(1-\alpha)(T+N-t-1) - \alpha + \alpha^{T+N-t}] \right\} \\
&= \frac{\alpha \Gamma_2^*}{1-\alpha} [(t-T)(1 - \alpha^N) + (\alpha^N - \alpha^{T+N-t}) N].
\end{aligned}$$

Since $\frac{1}{r} = \frac{\alpha}{1-\alpha}$, we then have

$$A_t = \frac{1 - \alpha^{T+N-t}}{1 - \alpha^N} A_T + \left(t - T + \frac{\alpha^N - \alpha^{T+N-t}}{1 - \alpha^N} N \right) \frac{\Gamma_2^*}{r}, \quad t \geq T, \quad (\text{A.8a})$$

In particular,

$$A_{T+1} = \frac{1 - \alpha^{N-1}}{1 - \alpha^N} A_T + \left(1 + \frac{\alpha^N - \alpha^{N-1}}{1 - \alpha^N} N \right) \frac{\Gamma_2^*}{r}. \quad (\text{A.8b})$$

For $t \leq T - 1$, by (A.7a),

$$A_t - A_{t-1} = \alpha^{T-t}(A_T - A_{T-1}) + \Gamma_1^* \sum_{i=1}^{T-t} \alpha^i = \alpha^{T-t}(A_T - A_{T-1}) + \frac{\alpha - \alpha^{T+1-t}}{1 - \alpha} \Gamma_1^*.$$

Then,

$$\begin{aligned} A_{t-1} &= A_t - (1 - \alpha^{T-t}) \frac{\Gamma_1^*}{r} - \alpha^{T-t}(A_T - A_{T-1}) \\ &= A_{t+1} - (1 - \alpha^{T-t-1}) \frac{\Gamma_1^*}{r} - \alpha^{T-t-1}(A_T - A_{T-1}) - (1 - \alpha^{T-t}) \frac{\Gamma_1^*}{r} - \alpha^{T-t}(A_T - A_{T-1}) \\ &= \dots \\ &= A_T - \frac{\Gamma_1^*}{r} \sum_{i=t}^T (1 - \alpha^{T-i}) - (A_T - A_{T-1}) \sum_{i=t}^T \alpha^{T-i} \\ &= A_T - \frac{\Gamma_1^* (1 - \alpha)T - \alpha - (1 - \alpha)t + \alpha^{T+1-t}}{1 - \alpha} - (A_T - A_{T-1}) \frac{1 - \alpha^{T+1-t}}{1 - \alpha}, \end{aligned}$$

implying

$$A_t = A_T - (A_T - A_{T-1}) \frac{1 - \alpha^{T-t}}{1 - \alpha} - \frac{(1 - \alpha)(T - t - 1) - \alpha + \alpha^{T-t} \Gamma_1^*}{1 - \alpha} \frac{\Gamma_1^*}{r}.$$

Then, by (A.7b),

$$A_t = A_T - \alpha(A_{T+1} - A_T) \frac{1 - \alpha^{T-t}}{1 - \alpha} - \frac{Y_0 + \Gamma_2^*}{r} (1 - \alpha^{T-t}) - \frac{(1 - \alpha)(T - t - 1) - \alpha + \alpha^{T-t} \Gamma_1^*}{1 - \alpha} \frac{\Gamma_1^*}{r},$$

i.e.,

$$A_t = A_T - \alpha(A_{T+1} - A_T) \frac{1 - \alpha^{T-t}}{1 - \alpha} - \frac{Y_0 + \Gamma_2^*}{r} (1 - \alpha^{T-t}) + \left(t - T + \frac{1 - \alpha^{T-t}}{1 - \alpha} \right) \frac{\Gamma_1^*}{r}, \quad t \leq T.$$

Substituting (A.8b) into this yields

$$\begin{aligned} A_t &= \frac{1 - \alpha^{T+N-t}}{1 - \alpha^N} A_T - (1 - \alpha^{T-t}) \frac{Y_0}{r} + \left(t - T + \frac{1 - \alpha^{T-t}}{1 - \alpha} \right) \frac{\Gamma_1^*}{r} \\ &\quad + \left(\frac{\alpha^N - \alpha^{T+N-t}}{1 - \alpha^N} N - \frac{1 - \alpha^{T-t}}{1 - \alpha} \right) \frac{\Gamma_2^*}{r}, \quad \text{for } t \leq T. \end{aligned} \quad (\text{A.8c})$$

In particular, for $t = 0$, (A.8c) becomes

$$A_0 = \frac{1 - \alpha^{T+N}}{1 - \alpha^N} A_T - (1 - \alpha^T) \frac{Y_0}{r} + \left(\frac{1 - \alpha^T}{1 - \alpha} - T \right) \frac{\Gamma_1^*}{r} + \left(\frac{1 - \alpha^T}{1 - \alpha^N} N \alpha^N - \frac{1 - \alpha^T}{1 - \alpha} \right) \frac{\Gamma_2^*}{r}. \quad (\text{A.9})$$

(A.8c) and (A.9) imply

$$\begin{aligned}
& (1 - \alpha^{T+N})A_t - (1 - \alpha^{T+N-t})A_0 \\
&= \frac{Y_0}{r}(\alpha^{T-t} - \alpha^T)(1 - \alpha^N) + [(1 - \alpha^{T+N})t - (\alpha^{T-t} - \alpha^T)(T + N)\alpha^N] \frac{\Gamma_1^*}{r} \\
&+ \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) (\alpha^{T-t} - \alpha^T) \frac{\Gamma_2^* - \Gamma_1^*}{r}. \tag{A.10}
\end{aligned}$$

Thus,

$$\begin{aligned}
\hat{A}_t &= \frac{1 - \alpha^{T+N-t}}{1 - \alpha^{T+N}} A_0 + \frac{(1 - \alpha^N)(\alpha^{T-t} - \alpha^T) Y_0}{1 - \alpha^{T+N}} \frac{1}{r} + \frac{\alpha^{T-t} - \alpha^T}{1 - \alpha^{T+N}} \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{r} \\
&+ \left[t - \frac{\alpha^{T+N-t} - \alpha^{T+N}}{1 - \alpha^{T+N}} (T + N) \right] \frac{\Gamma_1^*}{r}, \quad \text{for } t \leq T, \tag{A.11a}
\end{aligned}$$

As a special case of (A.11a), for $t = T$, we have

$$\begin{aligned}
\hat{A}_T &= \frac{1 - \alpha^N}{1 - \alpha^{T+N}} A_0 + \frac{(1 - \alpha^N)(1 - \alpha^T) Y_0}{1 - \alpha^{T+N}} \frac{1}{r} + \frac{1 - \alpha^T}{1 - \alpha^{T+N}} \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{r} \\
&+ \left[T - \frac{\alpha^N - \alpha^{T+N}}{1 - \alpha^{T+N}} (T + N) \right] \frac{\Gamma_1^*}{r}, \tag{A.11b}
\end{aligned}$$

Substituting (A.11b) into (A.8a) gives

$$\begin{aligned}
\hat{A}_t &= \frac{1 - \alpha^{T+N-t}}{1 - \alpha^{T+N}} A_0 + \frac{(1 - \alpha^T)(1 - \alpha^{T+N-t}) Y_0}{1 - \alpha^{T+N}} \frac{1}{r} + \frac{1 - \alpha^{T+N-t}}{1 - \alpha^{T+N}} \left(\frac{1 - \alpha^T}{1 - \alpha} - T \right) \frac{\Gamma_2^* - \Gamma_1^*}{r} \\
&+ \left[t - \frac{\alpha^{T+N-t} - \alpha^{T+N}}{1 - \alpha^{T+N}} (T + N) \right] \frac{\Gamma_2^*}{r}, \quad \text{for } t \geq T. \tag{A.11c}
\end{aligned}$$

We can also verify that (A.11b) is a special case of (A.11c).

Saving and Consumption:

By (A.11a), the saving for $t \leq T$ is

$$\begin{aligned}
\hat{S}_t &= \hat{A}_t - R\hat{A}_{t-1} = -\frac{r}{1 - \alpha^{T+N}} A_0 + \frac{\alpha^T - \alpha^{T+N}}{1 - \alpha^{T+N}} Y_0 + \frac{\alpha^T}{1 - \alpha^{T+N}} \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \Gamma_2^* \\
&+ \left[\frac{1}{1 - \alpha} - \frac{(T + N)\alpha^{T+N}}{1 - \alpha^{T+N}} - \frac{\alpha^T}{1 - \alpha^{T+N}} \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) - t \right] \Gamma_1^*.
\end{aligned}$$

Thus,

$$\hat{S}_t = -\frac{r}{1 - \alpha^{T+N}} A_0 + \frac{\alpha^T - \alpha^{T+N}}{1 - \alpha^{T+N}} Y_0 + \alpha^T \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{1 - \alpha^{T+N}} + (\bar{t}_\alpha - t) \Gamma_1^*, \quad t \leq T. \tag{A.12a}$$

By (A.11c), the saving for $t > T$ is

$$\begin{aligned}\hat{S}_t &= \hat{A}_t - R\hat{A}_{t-1} = -\frac{r}{1-\alpha^{T+N}}A_0 - \frac{1-\alpha^T}{1-\alpha^{T+N}}Y_0 - \frac{1}{1-\alpha^{T+N}}\left(T - \frac{1-\alpha^T}{1-\alpha}\right)\Gamma_1^* \\ &\quad + \left[\bar{t}_\alpha - t + \frac{1}{1-\alpha^{T+N}}\left(T - \frac{1-\alpha^T}{1-\alpha}\right)\right]\Gamma_2^*.\end{aligned}$$

Thus,

$$\hat{S}_t = -\frac{r}{1-\alpha^{T+N}}A_0 - \frac{1-\alpha^T}{1-\alpha^{T+N}}Y_0 + \left(T - \frac{1-\alpha^T}{1-\alpha}\right)\frac{\Gamma_2^* - \Gamma_1^*}{1-\alpha^{T+N}} + (\bar{t}_\alpha - t)\Gamma_2^*, \quad t > T. \quad (\text{A.12b})$$

By the budget constraint, the consumption is

$$\hat{C}_t = Y_t - \hat{S}_t = Y_t + \frac{r}{1-\alpha^{T+N}}A_0 - \frac{\alpha^T - \alpha^{T+N}}{1-\alpha^{T+N}}Y_0 - \alpha^T\left(\frac{1-\alpha^N}{1-\alpha} - N\alpha^N\right)\frac{\Gamma_2^* - \Gamma_1^*}{1-\alpha^{T+N}} + (t - \bar{t}_\alpha)\Gamma_1^*, \quad t \leq T; \quad (\text{A.13a})$$

$$\hat{C}_t = Y_t - \hat{S}_t = Y_t + \frac{r}{1-\alpha^{T+N}}A_0 + \frac{1-\alpha^T}{1-\alpha^{T+N}}Y_0 - \left(T - \frac{1-\alpha^T}{1-\alpha}\right)\frac{\Gamma_2^* - \Gamma_1^*}{1-\alpha^{T+N}} + (t - \bar{t}_\alpha)\Gamma_2^*, \quad t > T. \quad (\text{A.13b})$$

Since

$$\begin{aligned}t - \bar{t}_\alpha - \alpha^T\left(N\alpha^N - \frac{1-\alpha^N}{1-\alpha}\right)\frac{1}{1-\alpha^{T+N}} &= t - T - \frac{1}{1-\alpha^{T+N}}\left(\frac{1-\alpha^T}{1-\alpha} - T\right), \\ \alpha^T\left(N\alpha^N - \frac{1-\alpha^N}{1-\alpha}\right)\frac{1}{1-\alpha^{T+N}} &= \frac{1}{1-\alpha^{T+N}}\left(\frac{1-\alpha^T}{1-\alpha} + N\alpha^{T+N}\right) - \frac{1}{1-\alpha},\end{aligned}$$

we can further simplify (A.13a) and (A.13b) to (2). There is no consumption drop at retirement; in this case, a saving drop at retirement accommodates the income drop.

Welfare:

Let us now find the maximum utility. The Euler equation is, for all t ,

$$e^{-\theta C_t} = \beta E_{t-1} e^{-\theta C_{t+1}}.$$

Then,

$$e^{-\theta C_1} = \beta E_1 e^{-\theta C_2} = \beta^2 E_1 e^{-\theta C_3} = \dots = \beta^{t-1} E_1 e^{-\theta C_t}.$$

We then have

$$\begin{aligned} V(A_0) &= -E_0 E_1 \sum_{t=1}^{T+N} \frac{1}{\theta} e^{-\theta C_t} (\alpha\beta)^t = -\frac{1}{\theta} E_0 \sum_{t=1}^{T+N} (\alpha\beta)^t \beta^{1-t} e^{-\theta C_1} \\ &= -\frac{\alpha\beta}{\theta} \frac{1 - \alpha^{T+N}}{1 - \alpha} (E_0 e^{-\theta C_1}) = -\frac{\beta}{\theta r} (1 - \alpha^{T+N}) (E_0 e^{-\theta C_1}). \end{aligned} \quad (\text{A.14})$$

By (A.13a),

$$C_1 = Y_1 + \frac{r}{1 - \alpha^{T+N}} A_0 - \frac{\alpha^T - \alpha^{T+N}}{1 - \alpha^{T+N}} Y_0 - \alpha^T \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{1 - \alpha^{T+N}} + (1 - \bar{t}_\alpha) \Gamma_1^*.$$

Since $Y_1 = Y_0 + \xi_1 = Y_0 + \varepsilon_1$, we have

$$C_1 = \varepsilon_1 + b,$$

where

$$b \equiv \frac{r}{1 - \alpha^{T+N}} A_0 + \frac{1 - \alpha^T}{1 - \alpha^{T+N}} Y_0 - \alpha^T \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{1 - \alpha^{T+N}} + (1 - \bar{t}_\alpha) \Gamma_1^*. \quad (\text{A.15})$$

By the definition of Γ_1^* , we have

$$E_0 e^{-\theta C_1} = E_0 e^{-\theta[\varepsilon_1 + b]} = e^{-\theta b} E_0 e^{-\theta \varepsilon_1} = e^{-\theta b} e^{\frac{1}{2}\theta^2 \sigma_1^2} = e^{\theta(\frac{1}{2}\theta \sigma_1^2 - b)}.$$

Thus,

$$V(A_0) = -\frac{\beta}{\theta r} (1 - \alpha^{T+N}) e^{\theta(\frac{1}{2}\theta \sigma_1^2 - b)} = -\frac{1}{\theta r} (1 - \alpha^{T+N}) e^{\theta(\Gamma_1^* - b)}.$$

Substituting (A.15) into this then gives

$$V(A_0) = -\frac{1 - \alpha^{T+N}}{\theta r} e^{-\theta \left[\frac{r}{1 - \alpha^{T+N}} A_0 + \frac{1 - \alpha^T}{1 - \alpha^{T+N}} Y_0 - \alpha^T \left(\frac{1 - \alpha^N}{1 - \alpha} - N\alpha^N \right) \frac{\Gamma_2^* - \Gamma_1^*}{1 - \alpha^{T+N}} - \bar{t}_\alpha \Gamma_1^* \right]}. \quad (\text{A.16})$$

A.3. Equilibrium

Denote the solution in (3.2) as $A_t^*(A_0, Y_0)$, i.e., the person with initial wealth endowment a and permanent income y will have wealth $A_t^*(a, y)$ at age t . The total wealth for individuals i with endowment near (a_i, y_i) is

$$\sum_{t=1}^{T+N} A_t^*(a_i, y_i) \frac{1-p}{1-p^{T+N}} p^{t-1} f(a_i, y_i) \Delta a_i \Delta y_i.$$

The aggregate wealth is thus

$$W^* = \sum_i \left[\frac{1-p}{1-p^{T+N}} \sum_{t=1}^{T+N} p^{t-1} A_t^*(a_i, y_i) \right] f(a_i, y_i) \Delta a_i \Delta y_i,$$

which converges to $\int \frac{1-p}{1-p^{T+N}} \sum_{t=1}^{T+N} p^{t-1} A_t^*(a, y) da dy$ as Δa_i and Δy_i go to zero.

Thus,

$$W^* = \frac{1-p}{1-p^{T+N}} \sum_{t=1}^{T+N} p^{t-1} E(A_t^*). \quad (\text{A.17})$$

The equilibrium condition is

$$\frac{1-p}{1-p^{T+N}} \mu_{A_0} = (1-p)W. \quad (\text{A.18})$$

Substituting (A.11a), (A.11c) and (A.18) into (A.17) gives

$$\begin{aligned} W &= (1-p)W \sum_{t=1}^{T+N} p^{t-1} \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} \\ &+ \frac{1-p}{1-p^{T+N}} \frac{\mu_{Y_0}}{r} \left[\frac{1-\alpha^N}{1-\alpha^{T+N}} \sum_{t=1}^T p^{t-1} (\alpha^{T-t} - \alpha^T) + \frac{1-\alpha^T}{1-\alpha^{T+N}} \sum_{t=T+1}^{T+N} p^{t-1} (1-\alpha^{T+N-t}) \right] \\ &+ \frac{1-p}{1-p^{T+N}} \left\{ \sum_{t=1}^T p^{t-1} \left[t - \frac{\alpha^{T-t} - \alpha^T}{1-\alpha^{T+N}} \left(T\alpha^N + \frac{1-\alpha^N}{1-\alpha} \right) \right] + \sum_{t=T+1}^{T+N} p^{t-1} \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} \left(T - \frac{1-\alpha^T}{1-\alpha} \right) \right\} \frac{\Gamma_1^*}{r} \\ &+ \frac{1-p}{1-p^{T+N}} \left\{ \sum_{t=1}^T p^{t-1} \frac{\alpha^{T-t} - \alpha^T}{1-\alpha^{T+N}} \left(\frac{1-\alpha^N}{1-\alpha} - N\alpha^N \right) \right. \\ &\left. + \sum_{t=T+1}^{T+N} p^{t-1} \left[t - T - N + \frac{1-\alpha^{T+N-t}}{1-\alpha^{T+N}} \left(N + \frac{1-\alpha^T}{1-\alpha} \right) \right] \right\} \frac{\Gamma_2^*}{r}. \end{aligned}$$

This equation can be simplified substantially and then solved for the aggregate wealth in (3.8).

Appendix B:

Decomposition of Coefficient of Variation

Denote the constant correlation coefficient ρ between A_0 and Y_0 , $\rho = \frac{\text{cov}(A_0, Y_0)}{\sigma_{A_0} \sigma_{Y_0}}$.
By (3.7),

$$\text{var}(A_t^*) = a_t^2 \sigma_{A_0}^2 + 2a_t b_t \rho \sigma_{A_0} \sigma_{Y_0} + b_t^2 \sigma_{Y_0}^2. \quad (\text{B.1})$$

The expected value of A_t^* can be written as

$$E_t(A_t^*) = a_t(1 - p^{T+N})W^* + b_t \mu_{Y_0} + c_t. \quad (\text{B.2})$$

The CV within age group t and the CV between age group t and the national average wealth are:

$$CV_W(t) \equiv \frac{\sqrt{\text{var}(A_t^*)}}{E(A_t^*)}, \quad (\text{B.3})$$

$$CV_B(t) \equiv \frac{\sqrt{[E(A_t^*) - W^*]^2}}{E(A_t^*)}, \quad (\text{B.4})$$

and the total CV for age group t is

$$CV_T(t) \equiv \frac{\sqrt{E[(A_t^* - W^*)^2]}}{E(A_t^*)}. \quad (\text{B.5})$$

The square of the $CV(t)$ satisfies decomposability: $[CV_T(t)]^2 = [CV_B(t)]^2 + [CV_W(t)]^2$.

The same relationships can be defined for all cohorts together:

$$CV_B \equiv \frac{1}{W^*} \sqrt{\sum_{t=1}^{T+N} \frac{1-p}{1-p^{T+N}} p^{t-1} [E(A_t^*) - W^*]^2}, \quad (\text{B.6})$$

$$CV_W \equiv \frac{1}{W^*} \sqrt{\sum_{t=1}^{T+N} \frac{1-p}{1-p^{T+N}} p^{t-1} \text{var}(A_t^*)}, \quad (\text{B.7})$$

and the total CV is

$$CV_T \equiv \frac{1}{W^*} \sqrt{\sum_{t=1}^{T+N} \frac{1-p}{1-p^{T+N}} p^{t-1} E[(A_t^* - W^*)^2]}. \quad (\text{B.8})$$

Again, the square of the CV satisfies decomposability: $(CV_T)^2 = (CV_B)^2 + (CV_W)^2$.

Note that when all individual wealth holdings change by the same factor, CV_T , CV_B and CV_W are not affected (mean-invariant). This explains the presence of the multiplier $1/W^*$ in the definitions.

Similarly, for earnings, we can also define the CV within age group t , $CV_{W,Y}(t)$, and the CV between age group t and the national average earning, $CV_{B,Y}(t)$, as in (B.3) and (B.4). Notice that the earnings Y_t can be written as $Y_t = Y_0 + \sum_{i=1}^t \varepsilon_i$ for

$$t \leq T \text{ and } Y_t = \sum_{i=1}^t \varepsilon_i \text{ for } t > T.$$

For consumption, we can also define the CV within age group t , $CV_{W,C}(t)$, and the CV between age group t and the national average earning, $CV_{B,C}(t)$, as in (B.3) and (B.4). Notice that, similar to (3.7), we can write C_t in (3.4) as

$$C_t = \bar{a}_t A_0 + \bar{b}_t Y_0 + \bar{c}_t + \sum_{i=1}^t \varepsilon_i,$$

where \bar{a}_t , \bar{b}_t and \bar{c}_t are non-stochastic age-dependent constants, defined by (3.4).

Finally, for income, we can also define the CV within age group t , $CV_{W,I}(t)$, and the CV between age group t and the national average earning, $CV_{B,I}(t)$, as in (B.3) and (B.4). Notice that, similar to (3.7), the income can be written as

$$\begin{aligned} I_t &\equiv Y_t + rA_{t-1} = A_t - A_{t-1} + C_t \\ &= (a_t - a_{t-1})A_0 + (b_t - b_{t-1})Y_0 + c_t - c_{t-1} + \bar{a}_t A_0 + \bar{b}_t Y_0 + \bar{c}_t + \sum_{i=1}^t \varepsilon_i \\ &= (a_t - a_{t-1} + \bar{a}_t)A_0 + (b_t - b_{t-1} + \bar{b}_t)Y_0 + c_t - c_{t-1} + \bar{c}_t + \sum_{i=1}^t \varepsilon_i, \quad \text{for any } t. \end{aligned}$$

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