Robustness of a Fixed-Rent Contract in a Standard Agency Model

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Summary. It is well known that there are infinitely many incentive contracts that achieve the full information outcome in the standard agency model when the agent is risk-neutral. However, since Harris and Raviv (1979), the fixed-rent contract has been the focal point among those infinitely many first-best contracts. This paper examines whether the fixed-rent contract is robust or not in various circumstances.

Keywords and Phrases: Linear contract, Limiting contract, Robustness

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1. Introduction

In a standard principal-agent model, a single principal delegates to a single agent to act on his behalf. Since the principal cannot observe the agent's action choices directly, he has to make use of some observables such as outputs, annual earnings, and stock prices to motivate the agent to take appropriate actions when designing the agent's incentive contract.[1]

The standard principal-agent model has usually taken two different forms depending on how the agent's risk attitude is assumed. When the agent is risk-averse, the principal must take into account not only providing incentive for the agent but also optimally sharing risk with the agent for designing an incentive contract. But, there is a trade-off between these two concerns, and the full information outcome is not generally obtainable. In other words, only the second-best incentive outcome is available, and it is uniquely determined depending on the characteristics of the agent's risk preferences and his risk environments.

On the other hand, when the agent is risk-neutral, the full information outcome is easily obtainable. Intuitively, when both the principal and the agent are risk-neutral, the principal, in designing an incentive contract, has to consider how to provide the agent with incentive but does not have to consider how to share risks with the agent. Thus, as is shown in Harris and Raviv (1979), the principal can obtain the first-best outcome by charging the agent a fixed amount and giving him all the residuals. However, such a *fixed-rent* contract is not the only first-best contract, and there are indeed infinitely many other incentive contracts such as bonus contracts, option contracts, and non-linear contracts that also achieve the same outcome.

Since Harris and Raviv (1979), however, most agency papers have almost exclusively taken the fixed-rent contract as the optimal contract rather than any other first-best contract when the agent is risk-neutral. The fixed-rent contract is the only linear contract among the first-best contracts and it also contains several meaningful implications such as "the agent becomes a residual claimant" or "the agent vertically integrates the principal". However, before we take the fixed-rent contract for granted, we have to check whether the principal's choosing the fixed-rent contract over the other first-best contracts can always be justified, and, if not, under what circumstances it can be justified.

[1] For details of the standard principal-agent model, see Ross (1973), Stiglitz (1974), Mirrlees (1974), Harris and Raviv (1979), Holmstrom (1979), and Shavell (1979). Also, for a detailed survey of the agency literature, see Hart and Holmstrom (1987).

In this paper, we test the robustness of the fixed-rent contract by perturbing the standard framework in two different directions: the agent's risk preferences and his risk environments. We first examine the robustness of the fixed-rent contract in terms of the agent's risk preferences by investigating whether the unique optimal incentive contract when the agent is risk-averse converges to the fixed-rent contract as the agent's risk-aversion approaches zero (strong robustness) and whether the difference between the principal's expected payoff from the optimal contract and that from the linear contract approaches zero as the agent's risk-aversion goes to zero (weak robustness). Secondly, we examine its robustness in terms of the agent's risk environments by investigating whether it remains to be first-best as the risk environments change.

We first show that the form of limiting contract when the agent's risk-aversion converges to zero still depends on the characteristics of the agent's risk attitude and his risk environments. In other words, there are infinitely many limiting contracts depending on what convergence path (i.e., the agent's risk preferences and his risk environments) is actually taken. This suggests that the fixed-rent contract is not 'strongly robust' in terms of the agent's risk preferences and thus is not the best approximation for the optimal contract when the agent is almost risk-neutral. It also explains why there are suddenly many optimal contracts when the agent is risk-neutral, although there is only a unique optimal contract when the agent is risk-averse. Intuitively, each of the first-best contracts is a limiting contract of its own convergence path. But, we show that the difference between the principal's expected payoff from the optimal contract and that from the linear one approaches zero as the agent's risk-aversion goes to zero. This suggests that the fixed-rent contract is 'weakly robust' in terms of the agent's risk preferences and thus can be a good approximation for the optimal contract when the agent is almost risk-neutral.

On the other hand, the fixed-rent contract is the ONLY incentive contract that remains to be first-best when the agent's risk environments change, suggesting that the fixed-rent contract is indeed best among the first-best contracts when some parameters that affect the agent's risk environments are unknown to the principal or unstable.

The rest of the paper is organized as follows. In Section 2, our basic model is formulated. In Section 3, the first-best outcome is discussed assuming that the agent is risk-neutral. In Section 4, testing the robustness of the fixed-rent contract in terms of the agent's risk preferences is provided, whereas testing its robustness in terms of the agent's risk environments is provided in Section 5. Concluding remarks are given in Section 6, and all the proofs are provided in the Appendix.

2. The Basic Model

We consider a single period principal-agent model in which a risk-neutral agent works for a risk-neutral principal by investing his effort, $a \in \mathbb{A} \equiv [0,\ \overline{a}],\ \overline{a} < \infty,$ which is not directly observable to the principal. Output, $x \in \mathbb{X} \equiv [\underline{x},\ \overline{x}],\ \underline{x} > -\infty,\ \overline{x} < \infty,$ is determined not only by the agent's effort choice but also by the state of nature, $\theta,$ i.e., $x = X(a, \theta),$ and it becomes publicly observable at the end of the period. We use $f(x|a)$ to suppress $\theta,$ where $f \in \mathcal{F}$ denotes the output density function conditional on the agent's effort choice and $\mathcal{F}$ denotes the set of such density functions. After x is realized, the principal pays monetary wage s to the agent. Since x is the only observable, the agent's wage contract must be based on $x,$ i.e., $s = s(x).$ We assume that the agent's utility on monetary income and effort is additively separable such as

$$U(s, a) = u(s) - v(a), \tag{1}$$

where $u(\cdot)$ denotes the agent's utility on monetary income and $v(\cdot)$ denotes the agent's disutility of exerting $a.$

For analytical simplicity, we first make the following assumptions.

Assumption 1. $v'(a) \geq 0,\ v''(a) > 0, v(0) = 0, v'(0) = 0.$

Assumption 2. For any $x \in \mathbb{X},\ f(x|a)$ is twice differentiable in $a.$

Assumption 1 implies that the agent is work-averse in an increasing manner, whereas Assumption 2 is given to guarantee the existence of an optimal wage contract.

3. The First-Best Outcome

To discuss the full information outcome, we first assume that the agent is risk-neutral, i.e., the agent's utility on monetary income is linear such as

$$U(s, a) = s - v(a). \tag{2}$$

And, we consider the following set of *admissible contracts.*

$$\mathcal{S} \equiv \{s : \mathbb{R} \rightarrow \mathbb{R} | s \text{ is Lebesgue measurable}\},$$

where $\mathbb{R}$ denotes the "real" line.

Assumption 3. $R(a) \equiv \int x f(x|a)dx, \quad R'(\cdot) > 0, \quad R''(\cdot) < 0.$

Assumption 3 states that the expected output increases at a decreasing rate as the agent invests more effort. This assumption is needed to guarantee the existence of the first-best effort level.

Suppose that the principal can observe the agent's effort choice directly, and thus is able to enforce the agent's effort choice by designing a forcing contract. Then, the principal's optimization problem is:

$$\max_{s\in\mathcal{S},\, a\in\mathbb{A}} \quad \int [x - s(x)] f(x|a)dx + \int s(x) f(x|a)dx - v(a) \tag{3}$$
$$\text{s.t.} \quad (i)\ \underline{s} \leq s(x) \leq \overline{s}, \quad \forall x.$$

In the above optimization program, the principal maximizes the combined utilities of the principal and the agent, where the relative weight placed on the agent's benefit is given by 1 to rule out a corner solution. Thus, it is different from the traditional one in which the principal maximizes his own benefit given that the optimizing agent receives his reservation level of utility. However, it is easy to see that (3) generates the same characterization for the optimal wage contract as the traditional one does. The constraint in the above program indicates that the agent's wage contract should exist in a given interval $\mathbb{S} \equiv [\underline{s},\ \overline{s}]$. The existence of the lower bound on the contract can be justified by the agent's limited liability, whereas the existence of the upper bound can be justified by that of the principal. In fact, this limited liability constraint is not needed when the agent is risk-neutral. However, it is needed for the existence of an optimal wage contract when the agent is risk-averse.[2] Since, as will be shown in the next section, one of our main objectives is to analyze the situation in which the agent is almost risk-neutral (i.e., the agent's risk aversion approaches zero), we place such a limited liability constraint even when the agent is risk-neutral to maintain logical consistency.

The principal's optimization problem in (3) can be rewritten as:

$$\max_{s\in\mathcal{S},\, a\in\mathbb{A}} \quad R(a) - v(a) \tag{4}$$
$$\text{s.t.} \quad (i)\ \underline{s} \leq s(x) \leq \overline{s}, \quad \forall x.$$

Assumptions 1–3 guarantee that the socially efficient effort level, $a^* > 0$, satisfying

$$R'(a^*) = v'(a^*), \tag{5}$$

[2] This existence issue is well addressed in Mirrlees (1974). For detailed discussion, see Mirrlees (1974).

uniquely exists.

On the other hand, when the principal cannot directly observe the agent's effort choice, designing a forcing contract is not feasible. Instead, the principal must design an incentive contract that is conditioned on the observed output to motivate the agent to work hard. Thus, the agent's wage contract, $s(x)$, must satisfy the following incentive constraint.

$$\int s(x) f_a(x|a) dx = v'(a). \tag{6}$$

In specifying the above incentive compatibility constraint, we use the first-order approach which is sufficiently valid due to Assumptions 1–3.

Harris and Raviv (1979) show that the fixed-rent contract, $s(x) = x - B$, under which the agent pays a fixed rent, B, to the principal and takes all the remaining, achieves the full information outcome. This fixed-rent contract is sometimes interpreted as the principal's selling the firm to the agent or the agent's vertical integration of the principal. The fixed rent, B, would be uniquely determined by the agent's reservation level of utility in the traditional principal-agent framework in which the principal maximizes his own benefit given that the self-optimizing agent is receiving his reservation level of utility. However, in our joint maximization framework, B can be any constant as long as $s(\underline{x}) = \underline{x} - B \geq \underline{s}$ and $s(\overline{x}) = \overline{x} - B \leq \overline{s}$. Thus, to guarantee the existence of the first-best fixed-rent contract, we assume that

$$\overline{x} - \underline{x} < \overline{s} - \underline{s}. \tag{7}$$

As mentioned in the Introduction, most agency papers have adopted the fixed-rent contract as the optimal contract without any reservation when the agent is risk-neutral. However, this fixed-rent contract is not the only wage contract that achieves the first-best outcome in this case. In fact, any contract that satisfies

$$\int s(x) f_a(x|a^*) dx = v'(a^*) \tag{8}$$

achieves the same first-best outcome as long as $\underline{s} \leq s(x) \leq \overline{s}$, and it is easy to see that there are infinitely many other contracts satisfying (8).

Therefore, an interesting question is, "Will the fixed-rent contract be the only contract that survives some meaningful perturbations and thereby be robust among all the first-best contracts?" In the following two sections, we first test the robustness of the fixed-rent contract in terms of the agent's risk attitude by investigating whether the fixed-rent contract is a limiting contract when the agent's risk aversion approaches zero. Second, we test its robustness in terms of the output density function by investigating whether the fixed-rent contract remains to be first-best for any output density function.

4. Robustness Test Through the Agent's Risk Attitude

Suppose that the agent is risk-averse, and thus has an increasing and concave utility function on income, i.e., $u' > 0$ and $u'' < 0$ in (1). Since our main objective in this section is to see if the principal's using the fixed-rent contract has theoretical justification when the agent is almost risk-neutral, we denote the agent's utility on income as

$$u(s;\alpha) = s + \alpha\phi(s;\alpha), \ \phi'' < 0, \tag{9}$$

for $s \in \mathbb{S}$ and $\alpha \in \mathbb{R}^+$. Note that $u'' < 0$ is equivalent to $\phi''(s;\alpha) < 0$ for any given $\alpha > 0$. Thus, α in (9) captures the agent's risk-aversion in the sense that the agent becomes less risk-averse as α gets smaller. We investigate if the fixed-rent contract is a limiting contract of the sequence of the optimal wage contracts as $\alpha \to 0^+$ (strong robustness) and if the difference between the principal's expected payoff from the optimal contract and that from the linear contract approaches zero as $\alpha \to 0^+$ (weak robustness).

Assumption 4. (MLRP: Monotone Likelihood Ratio Property) For any a, $\frac{\partial log f(x|a)}{\partial a}$ is increasing in x.

Assumption 5. (CDFC: Convexity Distribution Function Condition) For any x, $F(x|a) \equiv \int^x f(t|a)dt$ is convex in a.

Assumption 6. $\phi(s;\alpha)$ is differentiable in s, and $\phi'(s;\alpha)$ converges to a finite number as $\alpha \to 0^+$ for any $s \in \mathbb{S}$.

Assumptions 4 and 5 are given to justify the use of the first-order approach in characterizing the optimal wage contract when the agent is risk-averse. They imply that the output function, $x = X(a,\theta)$, is increasing in a with a decreasing rate in a stochastic sense.[3]

[3]Assumptions 4 and 5 imply Assumption 3 but the converse is not true.

Let Φ be the set of functions ϕ that satisfy Assumption 6. We denote the limit of $\phi'(s;\alpha)$ as $\phi'(s)$, i.e., $\phi'(s;\alpha) \to \phi'(s)$ as $\alpha \to 0^+$ for each $s \in \mathbb{S}$. Since using the first-order approach is valid due to Assumptions 4 and 5,[4] the principal's maximization problem given α can be written as:

$$\max_{s\in\mathcal{S},\, a\in\mathbb{A}} \int [x - s(x)] f(x|a)dx + \int u[s(x);\alpha] f(x|a)dx - v(a) \tag{10}$$
$$\text{s.t.} \quad (i) \int u(s(x);\alpha) f_a(x|a)dx = v'(a)$$
$$(ii)\ \underline{s} \le s(x) \le \overline{s}, \quad \forall x.$$

As in the previous section, the principal maximizes the combined utilities of the principal and the agent, which is different from the traditional one. However, (10) also generates the same characterization for the optimal contract as the traditional one does as long as the agent's reservation utility level is properly selected in the traditional one. We place the same limited liability constraint implying that the agent's monetary wage must exist in a given interval $[\underline{s},\ \overline{s}]$. As mentioned earlier, this limited liability constraint is needed to guarantee the existence of an optimal wage contract especially when the agent is risk-averse.

Let $(a^o(\alpha), s^o(x;\alpha))$ be the optimal solution for the above optimization program. Then, solving the Euler equation of the above program gives

$$s^o(x;\alpha) = \begin{cases} \underline{s}, & \text{if } x < \underline{x}(\alpha), \\ \hat{s}(x;\alpha), & \text{if } \underline{x}(\alpha) \le x \le \overline{x}(\alpha), \\ \overline{s}, & \text{if } \overline{x}(\alpha) < x, \end{cases} \tag{11}$$

where $\hat{s}(x;\alpha)$ is determined by the Euler equation:

$$\frac{1}{u'[\hat{s}(x;\alpha);\alpha]} = 1 + \mu^o(\alpha)\frac{f_a}{f}(x|a^o(\alpha)), \tag{12}$$

and $\underline{x}(\alpha)$ and $\overline{x}(\alpha)$ are defined by

$$\frac{1}{u'(\underline{s})} = 1 + \mu^o(\alpha)\frac{f_a}{f}(\underline{x}(\alpha), a^o(\alpha)), \quad \frac{1}{u'(\overline{s})} = 1 + \mu^o(\alpha)\frac{f_a}{f}(\overline{x}(\alpha), a^o(\alpha)).$$

In the above equation, $\mu^o(\alpha)$ denotes the optimized Lagrangian multiplier for the agent's incentive constraint given α.

[4] Grossman and Hart (1983) and Rogerson (1985) show that MLRP and CDFC are sufficient for the validity of the first-order approach in the standard principal-agent framework.

Note that, when the agent is risk-neutral, the first-best fixed-rent contract exists in the given interval, $[\underline{s},\ \overline{s}]$, i.e., $\underline{s} \leq x - B \leq \overline{s}$ based on (7). Thus, to test the strong robustness of the fixed-rent contract in terms of the agent's risk-aversion, it will actually suffice to check whether $\hat{s}(x;\alpha)$ in (12) converges to a linear one or not as $\alpha \to 0^+$.

Proposition 1. *(Limiting Contract) Assume that Assumptions 1 - 6 hold for given $\phi(s;\alpha)$, and suppose that $\phi''(s;\alpha) \to \phi''(s)$ as $\alpha \to 0^+$ for any $s \in \mathcal{S}$. If there exists a limiting contract, $s^l(x)$, to which $s^o(x;\alpha)$ converges as $\alpha \to 0^+$, and if there exists a limiting second-best effort level, a^l, to which $a^o(\alpha)$ converges as $\alpha \to 0^+$, then the limiting contract is*

$$s^l(x) = \begin{cases} \underline{s}, & \text{if } x < \underline{x}^l, \\ \hat{s}(x), & \text{if } \underline{x}^l \leq x \leq \overline{x}^l, \\ \overline{s} & \text{if } \overline{x}^l < x, \end{cases} \tag{13}$$

where $\hat{s}(x)$ is determined by the limiting Euler equation:

$$\phi'[\hat{s}(x)] = b\frac{f_a}{f}(x|a^l), \tag{14}$$

for a certain constant $b < 0$, and $\underline{x}^l$ and $\overline{x}^l$ are defined by[5]

$$\phi'(\underline{s}) = b\frac{f_a}{f}(\underline{x}^l|a^l), \quad \phi'(\overline{s}) = b\frac{f_a}{f}(\overline{x}^l|a^l). \ \blacksquare$$

Proposition 1 shows that if the limiting contract, $s^l(x)$, exists, then it must satisfy (13) and (14). Therefore, the limiting contract, $s^l(x)$, is uniquely determined by equations (13) and (14) once the convergence path $(\phi(s), f(x|a))$ is defined. In other words, the actual form of the limiting contract depends on both $\phi(s)$ and $f(x|a)$.

Suppose that the agent's utility on income has a HARA form such as:

$$u(s) = \frac{1}{\beta - \alpha}\left[(\beta s + \gamma)^{1-\frac{\alpha}{\beta}} - 1\right], \quad \text{for } s \geq -\frac{\gamma}{\beta}, \tag{15}$$

where $\alpha, \beta, \gamma \in \mathbb{R}$. We assume that $\gamma = 1$ when $\alpha = 0$, and $\beta = 1$ when $\gamma = 0$. Then, we have

$$u'(s) = (\beta s + \gamma)^{-\frac{\alpha}{\beta}} \geq 0, \quad u''(s) = -\alpha(\beta s + \gamma)^{-\frac{\alpha}{\beta}-1}.$$

[5] If such an $\underline{x}^l$ does not exist, then $\underline{x}^l = \underline{x}$. Also, if such an $\overline{x}^l$ dose not exist, then $\overline{x}^l = \overline{x}$.

Therefore, we see that

$$\begin{array}{ll} u \text{ is linear: } u(s) = s & \text{if } \ \alpha = 0, \\ u \text{ is exponential: } u(s) = \frac{1}{\alpha}(1 - e^{-\alpha s}) & \text{if } \ \beta \to 0, \\ u \text{ is homothetic: } u(s) = \frac{1}{1-\alpha}(s^{1-\alpha} - 1) & \text{if } \ \gamma = 0. \end{array}$$

From (9), we have

$$\phi(s;\alpha) = \frac{1}{\alpha}\left\{\frac{1}{\beta - \alpha}\left[(\beta s + \gamma)^{1-\frac{\alpha}{\beta}} - 1\right] - s\right\}. \tag{16}$$

Therefore, we obtain

$$\phi'(s;\alpha) = \frac{(\beta s + \gamma)^{-\frac{\alpha}{\beta}} - 1}{\alpha}, \quad \text{and} \quad \phi''(s;\alpha) = -(\beta s + \gamma)^{-\frac{\alpha}{\beta}-1} < 0. \tag{17}$$

From (17), we see that

$$\lim_{\alpha \to 0} \phi'(s;\alpha) = \phi'(s) = -\frac{1}{\beta}\log(\beta s + \gamma) = -\frac{1}{\beta}\log(\beta s + 1), \tag{18}$$

indicating that Assumption 6 is satisfied under any HARA convergence. Especially, when $\beta \to 0$ (i.e., u is exponential), we have

$$\phi'(s) = -s. \tag{19}$$

Now, by using (14) and (18), we derive that the limiting contract under the HARA convergence satisfies

$$\hat{s}(x) = \frac{1}{\beta}\left[\exp\left(-b\beta\frac{f_a}{f}(x|a^l)\right) - 1\right]. \tag{20}$$

Especially, using (14) and (19), we obtain that the limiting contract under an exponential convergence satisfies

$$\hat{s}(x) = -b\frac{f_a}{f}(x|a^l). \tag{21}$$

Therefore, if the agent's utility on income takes an exponential form (i.e., the agent's utility exhibits constant absolute risk-aversion) and $\frac{f_a}{f}(x|a^l)$ is linear in x which is the case with many familiar families of density functions such as normal, gamma, and etc., the fixed-rent contract can be 'strongly robust' because it is a limiting contract as the agent's risk-aversion approaches zero.

However, in general, the set of $(\phi(s), f(x|a))$ that produce the fixed-rent contract as a limiting contract will have measure zero compared with the whole space, $(\Phi, \mathcal{F})$. This indicates that the fixed-rent contract is NOT generally 'strongly robust' in terms of the agent's risk attitude, suggesting that the fixed-rent contract is not the best approximation for the optimal contract when the agent is almost risk-neutral. In other words, when the agent is almost risk-neutral, there usually exists a first-best contract which is a better approximation for the optimal contract than the fixed-rent contract. In addition, this result also explains why there are suddenly infinitely many first-best contracts when the agent is risk-neutral, whereas there is only a unique second-best contract when the agent is risk-averse. There are infinitely many different convergences [i.e., infinitely many different $(\phi(s), f(x|a))$], and the actual from of the limiting contract depends on which $(\phi(s), f(x|a))$ is taken. Thus, it can be well conjectured that each of the first-best contracts with the agent's being risk-neutral corresponds to its own convergence path $(\phi(s), f(x|a))$.

The above result is summarized in the following corollary.

Corollary 1. *Given Assumptions 1–6, the fixed-rent contract can be 'strongly robust' in terms of the agent's risk attitude if the agent's utility on income shows constant absolute risk-aversion (i.e., $u(s)$ is exponential) and $\frac{f_a}{f}(x|a)$ is linear in x. However, it is usually not a limiting contract for a general convergence.* ■

Note that the results in Propositions 1 and Corollary 1 are provided based on the assumption that the limiting contract, $s^l(x)$, and the limiting second-best effort level, a^l, exist as the agent's risk-aversion approaches zero. Thus, to complete our analysis, we need to show that they actually exist. The following proposition proves that.

Proposition 2. *Assume that Assumptions 1–6 hold for given $\phi(s;\alpha)$, and suppose that $\phi''(s;\alpha) \to \phi''(s)$ as $\alpha \to 0^+$ for any $s \in S$. Then, as $\alpha \to 0^+$, we have*

(a) *(Existence) there exist a limiting contract $s^l(x)$ to which $s^o(x;\alpha)$ converges, and a limiting effort a^l to which $a^o(\alpha)$ converges.*

(b) *(Optimality) the limiting contract and effort must be the first-best, and*

(c) *(Uniqueness) the limiting contract is unique.* ■

Proposition 2(a) states that both the limiting contract and the limiting second-best effort level actually exist, and Proposition 2(b) states that both the agent's second-best effort level given α, $a^o(\alpha)$, and the second-best wage contract given α, $s^o(x;\alpha)$, indeed converge to the first-best effort level, a^*, and one of the first-best wage contracts respectively as α converges to zero. Furthermore, Proposition 2(c) shows that the limiting contract, $s^l(x)$, characterized by (13), is unique given the convergence path $(\phi(s), f(x|a))$.

Based on Proposition 2, we can now investigate if the fixed-rent contract is 'weakly robust' in terms of the agent's risk attitude and derive the following corollary.

Corollary 2. *For any* $\phi(s;\alpha)$ *and for any given* $\delta > 0$, *there always exists* $\hat{\alpha} > 0$ *such that,* $\forall\alpha \in (0,\ \hat{\alpha})$,

$$\begin{aligned}
&\int [x - s^o(x;\alpha)]f(x|a^o(\alpha))dx + \int u[s^o(x;\alpha);\alpha]f(x|a^o(\alpha))dx - v(a^o(\alpha)) \\
&-\left\{\int [x-(x-B)]f(x|a^B(\alpha))dx + \int u(x-B;\alpha)f(x|a^B(\alpha))dx - v(a^B(\alpha))\right\} \\
&< \delta,
\end{aligned}$$

where $a^B(\alpha)$ *is the agent's effort level that will be induced by the fixed-rent contract given* α. ■

Corollary 2 shows that the difference between the joint benefit (or equivalently the principal's expected payoff guaranteeing the agent the reservation utility level) from the optimal contract, $s^o(x;\alpha)$, and that from the linear contract approaches zero as the agent's risk-aversion goes to zero. Thus, it indicates that the fixed-rent contract is 'weakly robust' in terms of the agent's risk attitude suggesting that, although the fixed-rent contract is not the best approximation for the optimal contract when the agent is almost risk-neutral, it can at least be a *good* approximation for the optimal contract in this case. Therefore, the principal's designing the fixed-rent contract as an approximation when the agent is almost risk-neutral can be weakly justified.

5. Robustness Test Through the Output Density Function

We now test the robustness of the fixed-rent contract in terms of the output density function, $f(x|a)$. Thus, in this section, we assume that the agent is risk-neutral. Instead, we consider the case in which the output density function varies, and investigate if the fixed-rent contract remains to be first-best.

As the output density function changes, the first-best effort level may also change. Thus, without loss of any generality, we only consider the set of output density functions that preserve a^* as the first-best effort level, i.e.,

$$\mathcal{F}(a^*) \equiv \left\{ f(x|a) \in \mathcal{F} \,\middle|\, \int x f_a(x|a^*)dx = v'(a^*) \right\}. \tag{22}$$

Then, it is obvious that, for any output density function that preserves a^* as the first-best effort level, the fixed-rent contract $s(x) = x - B$ actually induces the agent to take a^* and thus remains to be the first-best contract because

$$\int s(x)f_a(x|a^*)dx = \int (x-B)f_a(x|a^*)dx = v'(a^*), \ \forall \ f \in \mathcal{F}(a^*).$$

However, it is still early to conclude that the fixed-rent contract must be chosen over any other first-best contract in this case. In fact, we have to examine whether there is any other wage contract that also remains to be first-best even if the output density function varies within $\mathcal{F}(a^*)$. The following proposition gives an answer to this question.

Proposition 3. *Assume that Assumptions 1–3 hold. Then, the fixed-rent contract, $s(x) = x - B$, where B is a constant, is the ONLY contract that remains as the first-best contract for any output density function in $F(a^*)$.* ■

Proposition 3 indicates that although there are infinitely many first-best contracts when the agent is risk-neutral, only the fixed-rent contract is stationary in the sense that it is invariant with the agent's risk environments, whereas all other first-best contracts are sensitive to the changes in risk environments. This result suggests that the fixed-rent contract is indeed the best wage contract among the first-best contracts when the agent's risk environments are unstable or uncertain. For instance, suppose that the output density function depends not only on the agent's effort choice, a, but also on some other factors (k) that are unknown both to the principal and the agent when they agree upon the contract but the agent can observe the true value of k before he makes his effort. Proposition 3 actually suggests that the fixed-rent contract is the only wage contract that achieves the first-best outcome regardless of k, whereas the outcome resulting from any other first-best contract depends on k.

6. Conclusion

Since Harris and Raviv (1979), most agency literature has adopted the fixed-rent contract as the first-best contract without any reservation when the agent is risk-neutral. However, the fixed-rent contract is not the only contract that achieves the full information outcome in this case. In fact, there are infinitely many other contracts that also obtain the same first-best outcome when the agent is risk-neutral. Thus, it is meaningful to study under what conditions such a fixed-rent contract dominates over the other first-best contracts.

In this paper, we investigate the robustness of the fixed-rent contract in two different aspects: the agent's risk preferences and his risk environments. We show that the fixed-rent contract is not generally a limiting contract of the sequence of the optimal contracts when the agent's risk aversion approaches zero. This result implies that the fixed-rent contract is not 'strongly robust' in terms of the agent's risk preferences, suggesting that it is not the best approximation for the optimal contract when the agent is almost risk-neutral. But, as we show, the difference between the principal's expected payoff from the optimal contract and that from the linear contract reduces to zero as the agent's risk-aversion approaches zero, suggesting that the fixed-rent contract is a good approximation for the optimal contract when the agent is almost risk-neutral. On the other hand, we show that the fixed-rent contract is the only contract that remains to be first-best even if the agent's risk environments change, suggesting that it is best among the first-best contracts when the risk environments are uncertain or unstable.

Appendix: Proofs of Propositions

Proof of Proposition 1

Define $h(x|a) \equiv \frac{f_a}{f}(x|a)$. With $u(s) = s + \alpha\phi(s;\alpha)$, (12) becomes

$$1 + \alpha\phi'[\hat{s}(x;\alpha);\alpha] = \frac{1}{1 + \mu^o(\alpha)h[x|\alpha^o(\alpha)]}. \tag{A1}$$

Then, we have

$$\frac{-\alpha\phi'[\hat{s}(x;\alpha);\alpha]}{1 + \alpha\phi'[\hat{s}(x;\alpha);\alpha]} = \mu^o(\alpha)h[x|a^o(\alpha)]. \tag{A2}$$

Thus, by differentiating both sides of (A2) with respect to x, we derive

$$\frac{-\alpha\phi''[\hat{s}(x;\alpha);\alpha]\hat{s}'(x;\alpha)}{\{1 + \alpha\phi'[\hat{s}(x;\alpha);\alpha]\}^2} = \mu^o(\alpha)h'[x|a^o(\alpha)]. \tag{A3}$$

Hence, we can use (A2) and (A3) to eliminate $\mu^o(\alpha)$ and obtain

$$\frac{\phi'[\hat{s}(x;\alpha);\alpha]\{1 + \alpha\phi'[\hat{s}(x;\alpha);\alpha]\}}{\phi''[\hat{s}(x;\alpha);\alpha]\hat{s}'(x;\alpha)} = \frac{h[x|a^o(\alpha)]}{h'[x|a^o(\alpha)]}.$$

Thus, we have

$$1 + \alpha\phi'[\hat{s}(x;\alpha);\alpha] = \frac{\phi''[\hat{s}(x;\alpha);\alpha]\hat{s}'(x;\alpha)}{\phi'[\hat{s}(x;\alpha);\alpha]}\frac{h[x|a^o(\alpha)]}{h'[x|a^o(\alpha)]}.$$

Since $\hat{s}(x;\alpha) \to \hat{s}(x)$ and $a^o \to a^l$ as $\alpha \to 0^+$, letting $\alpha \to 0^+$ gives

$$\frac{\phi''[\hat{s}(x)]\hat{s}'(x)}{\phi'[\hat{s}(x)]}\frac{h(x|a^l)}{h'(x|a^l)} = 1,$$

implying that

$$\frac{dh(x|a^l)}{h(x|a^l)} = \frac{d\phi'[\hat{s}(x)]}{\phi'[\hat{s}(x)]}.$$

The general solution for this differential equation is

$$\log h(x|a^l) + c = \log \phi'[\hat{s}(x)],$$

where c is an arbitrary constant. Therefore, the limiting contract $\hat{s}(x)$ satisfies equation (14) for a certain constant b. As we know, a decreasing contract cannot possibly be optimal. Thus, under Assumption 3, since ϕ' is decreasing in x and h is increasing in x, we must have $b < 0$.

Proof of Proposition 2

Part (a): (Existence) For any convergent sequence $\alpha_n \in \mathbb{R}^+$ with limit 0, consider the corresponding sequence of the optimal contracts $\{s^o(x;\alpha_n)\}_{n=1}^{\infty}$ and the corresponding sequence of the second-best effort levels $\{a^o(\alpha_n)\}_{n=1}^{\infty}$. Let $\{s^o(x;\alpha_{n_k})\}_{k=1}^{\infty}$ be a convergent subsequence of $\{s^o(x;\alpha_n)\}_{n=1}^{\infty}$ with $s^o(x;\alpha_{n_k}) \to s_0(x)$ almost surely as $k \to \infty$, and $\{a^o(\alpha_{n_k})\}_{k=1}^{\infty}$ be a convergent subsequence of $\{a^o(\alpha_n)\}_{n=1}^{\infty}$ with $a^o(\alpha_{n_k}) \to a_0$ as $k \to \infty$, where $s_0(x) \in [\underline{s},\ \overline{s}]$ and $a_0 \in [\underline{a},\ \overline{a}]$.

Since the first-order condition for a is

$$\int [x - s^o(x;\alpha)] f_a dx + \mu^o(\alpha)\left[\int u[s^o(x;\alpha)] f_{aa} dx - v''[a^o(\alpha)]\right] = 0,$$

and since $\mu^o(\alpha) \to 0$ as $\alpha \to 0^+$ from (12), and $\{s^o(x;\alpha_{n_k})\}$ and $\{a^o(\alpha_{n_k})\}$ are bounded, we have

$$\int [x - s^o(x;\alpha_{n_k})] f_a[x|a^o(\alpha_{n_k})] dx \to 0, \quad \text{as } k \to \infty.$$

This implies that

$$\int s^o(x;\alpha_{n_k}) f_a[x|a^o(\alpha_{n_k})] dx \to R'(a_0), \quad \text{as } k \to \infty.$$

Since the incentive compatibility constraint implies

$$\int u[s^o(x;\alpha_{n_k})] f_a[x|a^o(\alpha_{n_k})] dx = v'[a^o(\alpha_{n_k})],$$

and since in the limit $u(s) \to s$ for any s, we have

$$\int s_0(x) f_a(x|a_0) dx = v'(a_0). \tag{A4}$$

Therefore, we derive

$$R'(a_0) = v'(a_0). \tag{A5}$$

Now, by replacing " $\hat{s}(x;\alpha) \to \hat{s}(x)$ as $\alpha \to 0^+$ " by " $s^o(x;\alpha_{n_k}) \to s_0(x)$ as $k \to \infty$ ", and " $a^o(\alpha) \to a^l$ as $\alpha \to 0^+$ " by " $a^o(\alpha_{n_k}) \to a_0$ as $k \to \infty$ ", we can show from Proposition 1 that the limiting contract $s_0(x)$ satisfies

$$\phi'[s_0(x)] = b(a_0) h(x|a_0), \tag{A6}$$

for x such that $\underline{s} \le (\phi')^{-1}[b(a_0)h(x|a_0)] \le \overline{s}$, and $s_0(x)$ is $\underline{s}$ or $\overline{s}$ otherwise.

Equations (A4) and (A6) imply that

$$\int_{\underline{x}}^{\underline{x}^l} \underline{s} f_a(x|a_0)dx + \int_{\underline{x}^l}^{\overline{x}^l} (\phi')^{-1}[bh(x|a_0)] f_a(x|a_0)dx + \int_{\overline{x}^l}^{\overline{x}} \overline{s} f_a(x|a_0)dx = v'(a_0), \quad \text{(A7)}$$

where $\underline{x}^l$ and $\overline{x}^l$ are defined in Proposition 1 for given a_0. Since $\phi'' < 0$, by differentiating the left hand side of equation (A7) with respect to b, we have

$$\frac{d}{db}\left\{\int_{\underline{x}}^{\underline{x}^l} \underline{s} f_a(x|a_0)dx + \int_{\underline{x}^l}^{\overline{x}^l} (\phi')^{-1}[bh(x|a_0)] f_a(x|a_0)dx + \int_{\overline{x}^l}^{\overline{x}} \overline{s} f_a(x|a_0)dx\right\}$$

$$= \int_{\underline{x}^l}^{\overline{x}^l} \frac{h(x|a_0)}{\phi''[(\phi')^{-1}(bh(x|a_0))]} f_a(x|a_0)dx = \int_{\underline{x}^l}^{\overline{x}^l} \frac{h^2(x|a_0)}{\phi''[s(x)]} f(x|a_0)dx < 0.$$

This implies that b in (A6) must be unique. That is, for each a_0, there is a unique b that satisfies (A6), and the function $b(a_0)$ is well defined.

Since (A5) determines a unique solution, we have shown that the limit of any convergent subsequence of $\{a^o(\alpha_n)\}_{n=1}^{\infty}$ must all be the same. Thus, sequence $\{a^o(\alpha_n)\}_{n=1}^{\infty}$ itself must be convergent and its limit must satisfy (A5). In other words, $a^o(\alpha)$ must be convergent as $\alpha \to 0^+$ and the limit must also satisfy (A5). Furthermore, since the limiting contract satisfies (A6) with a unique limit a_0 for any subsequence $\{s^o(x;\alpha_{n_k})\}_{k=1}^{\infty}$, and since (A6) determines a unique contract for any given constant a_0 and a_0 is known to be uniquely determined by (A5), the limiting contract is unique as well. Therefore, any sequence $\{s^o(x;\alpha_n)\}_{n=1}^{\infty}$ converges to a unique contract, and as a result $s^o(x;\alpha)$ converges as $\alpha \to 0^+$ for any x.

Part (b): (Optimality) From Part (a), we know that the limiting effort satisfies (A5), and it in fact determines the first-best effort level. That is, the limiting effort must be the first-best.

Also, at risk neutrality $\alpha = 0$, the problem is

$$\begin{aligned} \max_{s \in \mathcal{S},\, a \in \mathbb{A}} \quad & R(a) - v(a) \\ \text{s.t.} \quad & \int s(x) f_a(x|a)dx = v'(a) \\ & \underline{s} \le s(x) \le \overline{s}. \end{aligned}$$

This problem is composed of two parts:

1. Choose optimal a^* satisfying

$$\max_{a\in\mathbb{A}} R(a) - v(a).$$

2. Any contract $s(x)$, satisfying $\int s(x)f_a(x|a^*)dx = v'(a^*)$ and $\underline{s} \le s(x) \le \overline{s}$, is optimal.

Thus, the conditions for $s(x)$ are

$$\begin{aligned}
&\int s(x)f_a(x|a^*)dx = v'(a^*), \\
&R'(a^*) = v'(a^*), \\
&\underline{s} \le s(x) \le \overline{s}.
\end{aligned}$$

By comparing the above two equations with (A4) and (A5), we know that the limiting contract must be the first-best as well.

Part (c): (Uniqueness) Part (a) has already proven the uniqueness of a^l and $s^l(x)$.

Proof of Corollary 2

Since $(s^o(x;\alpha), a^o(\alpha))$ is the optimal solution for the principal's optimization program when the agent is risk-averse with α, we know that

$$\begin{aligned}
&\int [x - s^o(x;\alpha)]f(x|a^o(\alpha))dx + \int u[s^o(x;\alpha);\alpha]f(x|a^o(\alpha))dx - v(a^o(\alpha)) \\
> &\int [x - (x-B)]f(x|a^B(\alpha))dx + \int u[x - B;\alpha]f(x|a^B(\alpha))dx - v(a^B(\alpha)).
\end{aligned}$$

In the above inequality, the left-hand side denotes the joint benefit when the optimal contract, $s^o(x;\alpha)$, is designed, whereas the right-hand side denotes the joint benefit when the fixed-rent contract, $s(x) = x - B$, is designed. Since $u(s;\alpha) \to s$ and $a^o(\alpha) \to a^*$ as $\alpha \to 0^+$ (by Proposition 2(b)), we have

$$\begin{aligned}
&\lim_{\alpha\to 0^+} \left\{ \int [x - s^o(x;\alpha)]f(x|a^o(\alpha))dx + \int u[s^o(x;\alpha);\alpha]f(x|a^o(\alpha))dx - v(a^o(\alpha)) \right\} \\
&= R(a^*) - v(a^*).
\end{aligned}$$

Also, since $a^B(\alpha) \to a^*$ as $\alpha \to 0^+$, we have

$$\lim_{\alpha \to 0^+} \left\{ \int [x-(x-B)] f(x|a^B(\alpha))dx + \int u(x-B;\alpha) f(x|a^B(\alpha))dx - v(a^B(\alpha)) \right\}$$
$$= R(a^*) - v(a^*).$$

Thus, by continuity, we can derive that, for any given $\delta > 0$, there always exists $\hat{\alpha} > 0$ such that for any $\alpha \in (0, \hat{\alpha})$

$$\int [x - s^o(x;\alpha)] f(x|a^o(\alpha))dx + \int u[s^o(x;\alpha);\alpha] f(x|a^o(\alpha))dx - v(a^o(\alpha))$$
$$- \left\{ \int [x-(x-B)] f(x|a^B(\alpha))dx + \int u(x-B;\alpha) f(x|a^B(\alpha))dx - v(a^B(\alpha)) \right\}$$
$$< \delta.$$

Proof of Proposition 3

Assume contrarily that there exists a contract $s(\cdot)$, where $s'(x) \neq 1$ for some x, that satisfies

$$\frac{\partial}{\partial a} \int s(x) f(x|a)dx \Big|_{a=a^*} = v'(a^*), \tag{A8}$$

and

$$\frac{\partial}{\partial a} \int x f(x|a)dx \Big|_{a=a^*} = v'(a^*), \tag{A9}$$

for any density function $f(x|a)$ satisfying Assumption 3. Now, let $\psi(x) \equiv s(x) - s^*(x)$, where $s^*(x) = x - B$. Then, (A8) and (A9) imply

$$\frac{\partial}{\partial a} \int \psi(x) f(x|a)dx \Big|_{a=a^*} = 0,$$

for any density function satisfying Assumption 3. Thus, to derive that $\psi'(x) \neq 0$ for some x is a contradiction, we have to show that there exists a distribution function $G(x|a)$ with density $g(x|a)$ satisfying:

(a) $g(x|a)$ satisfies Assumption 3,

(b) G is a^* preserving, i.e., $\frac{\partial}{\partial a} \int x g(x|a)dx \big|_{a=a^*} = v'(a^*)$,

(c) $\frac{\partial}{\partial a} \int \psi(x) g(x|a)dx \big|_{a=a^*} \neq 0$.

We divide the proof into two steps. In the first step, we assume $s(x)$ to be right-continuously differentiable. In the second step, we extend the proof to the case in which $s(x)$ has a few jumps. Also, we prove only for right-continuous derivatives.

Step 1: Continuous with a Right-Continuous Derivative

Consider a particular density function $f(x|a)$ that satisfies $f \in \mathcal{F}(a^*)$ and

$$R_f(a) \equiv \int x f(x|a)dx, \quad R_f(0) \geq 0, \quad R_f(\infty) < \infty, \quad R_f'(\cdot) > 0, \quad R_f''(\cdot) < 0. \tag{A10}$$

We assume that $\psi(x)$ is differentiable with a right-continuous derivative, and let

$$h(a, \varepsilon) = 2\varepsilon \left[R_f \left(\frac{a - a^*}{\varepsilon} + a^* \right) - R_f \left(\frac{\varepsilon - 1}{\varepsilon} a^* \right) \right], \tag{A11}$$

where $R_f(\cdot)$ is defined in (A10) for given density function $f(x|a)$. Also, assume that $\psi'(x_0) > 0$ at x_0. Note that, for any given $\varepsilon > 0$, $h(a, \varepsilon) \geq 0$, $\forall\, a \in \mathbb{A}$. Thus, we can consider the following uniform density function on $(x_0,\ x_0 + h(a, \varepsilon))$ for given $\varepsilon > 0$ such that

$$g(x|a) = \begin{cases} \frac{1}{h(a,\varepsilon)} & \text{for } \ x_0 < x < x_0 + h(a, \varepsilon) \\ 0 & \text{otherwise.} \end{cases}$$

1. Since

   $$\int x g(x|a)dx \equiv R_g(a) = x_0 + \varepsilon \left[R_f(\frac{a - a^*}{\varepsilon} + a^*) - R_f(\frac{\varepsilon - 1}{\varepsilon} a^*) \right],$$

   we can easily see that $g(x|a)$ satisfies Assumption 3. So, condition (a) is satisfied.

2. The a^* preserving property requires

   $$\frac{\partial}{\partial a} \int x g(x|a)dx \bigg|_{a=a^*} = v'(a^*),$$

   which is

   $$\frac{\partial}{\partial a} \left[x_0 + \frac{h(a, \varepsilon)}{2} \right]_{a=a^*} = v'(a^*),$$

   requiring

   $$h_a(a^*, \varepsilon) = 2v'(a^*).$$

   From (A11), we can easily see that the above equation is satisfied. So, condition (b) is also satisfied.

3. Note that $h(a^*, \varepsilon) \to 0$ as $\varepsilon \to 0$. Thus, since $\psi'(x)$ is right-continuous at x_0, when ε is sufficiently small, we have $\psi'(x) > 0$ for $x \in (x_0,\ x_0 + h(a^*, \varepsilon))$. Consider

$$
\begin{aligned}
\frac{\partial}{\partial a}\int \psi(x)g(x|a)dx &= \frac{\partial}{\partial a}\int_{x_0}^{x_0+h(a,\varepsilon)} \psi(x)\frac{1}{h(a,\varepsilon)}dx \\
&= \frac{h_a(a,\varepsilon)}{h(a,\varepsilon)}\psi[x_0 + h(a,\varepsilon)] - \frac{h_a(a,\varepsilon)}{[h(a,\varepsilon)]^2}\int_{x_0}^{x_0+h(a,\varepsilon)} \psi(x)dx \\
&= \frac{h_a(a,\varepsilon)}{h(a,\varepsilon)}\{\psi[x_0 + h(a,\varepsilon)] - \psi[x_0 + \theta h(a,\varepsilon)]\},
\end{aligned}
$$

for some $\theta \in (0,\ 1)$. Here, since $\psi(x)$ is continuous in $[x_0,\ x_0 + h(a^*, \varepsilon)]$, the mean-value theorem has been applied to find $\theta \in (0,\ 1)$. Since $\psi'(x) > 0$ on $(x_0,\ x_0 + h(a, \varepsilon))$, the above is strictly positive when $a = a^*$, i.e., condition (c) is satisfied.

Therefore, if $\psi'(x_0) \neq 0$ at an arbitrary point x_0, we can find a distribution function $G(x|a)$ such that conditions (a),(b), and (c) are satisfied. Thus, we must have $\psi'(x) = 0$ at any point, which implies $s'(x) = 1$ at any point.

Step 2: Piecewise Continuous with a Right-Continuous Derivative

Assume that except on finite points $x_1, ..., x_n$, $\psi(x)$ is continuous and differentiable with a right-continuous derivative.

Using $h(a, \varepsilon)$ defined in (A11), define

$$
g(x|a) = \begin{cases} \frac{1}{h(a,\varepsilon)+\varepsilon} & \text{for } x_0 - \varepsilon < x < x_0 + h(a,\varepsilon) \\ 0 & \text{otherwise,} \end{cases}
$$

implying

$$
G(x|a) = \begin{cases} 1 & \text{if } x \geq x_0 + h(a,\varepsilon) \\ \frac{x-x_0+\varepsilon}{h(a,\varepsilon)+\varepsilon} & \text{if } x_0 - \varepsilon < x < x_0 + h(a,\varepsilon) \\ 0 & \text{if } x \leq x_0 - \varepsilon. \end{cases}
$$

Conditions (a) and (b) are still satisfied. For condition (c), let $\Delta \equiv \psi(x_0^+) - \psi(x_0^-) \neq 0$. If ε is small enough such that in $(x_0 - \varepsilon,\ x_0 + h(a, \varepsilon))$ only x_0 belongs to $\{x_1, .., x_n\}$, then by the proof in Step 1, we have $\psi'(x) = 0$ except at x_0. Then,

$$\frac{\partial}{\partial a} \int_{-\infty}^{\infty} \psi(x) g(x|a) dx \bigg|_{a=a^*} = -G_a(x_0|a^*)\Delta.$$

Since we have

$$G_a(x_0|a^*) = -\frac{2\varepsilon R_f'(a^*)}{[2\varepsilon R_f(a^*) + 2\varepsilon R_f(\frac{\varepsilon-1}{\varepsilon}a^*) + \varepsilon]^2} \neq 0,$$

we can easily see that condition (c) is also satisfied. Hence, any jump-up or -down cannot exist.

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